Emergency units and time dependent queueing games
@drvinceknight
\[
\left(\begin{array}{cc}
(2, 2) & (5, 0) \\
(0, 5) & (4, 4)
\end{array}\right)
\]
\( k = 1 \)

\( \mathcal{P}_1 = \{1, 2\} \)

\( c_1 = 1 \) and \( c_2 = x \)

\( r = 1 \)

The Nash flow minimises:

\[
\phi(y, 1 - y) = \sum_{e=1}^{2} \int_{0}^{f_e} c_e(x) \, dx = \int_{0}^{y} 1 \, dx + \int_{0}^{1-y} x \, dx
\]

\[
= y + \frac{(1 - y)^2}{2} = \frac{1}{2} + \frac{y^2}{2}
\]

\( \Rightarrow \tilde{f} = (0, 1) \)
\[ k = m \]
\[ |\mathcal{P}_i| = n + 1 \]
\[ r_i = \Lambda_i \]
- \( k = m \)
- \(|\mathcal{P}_i| = n + 1\)
- \( r_i = \Lambda_i \)
Theorem: Assuming \( \sum_{i=1}^{m} \Lambda_i < \sum_{j=1}^{n} c_j \mu_j \) we have:

\[
\lim_{\beta_i \to \infty} PoA(\beta) < \infty \text{ for all } i \in [m]
\]

The price of anarchy increases with worth of service, up to a point.

Proof.

- \( \lim_{\beta_i \to \infty} \lambda^* = k^* \) and \( \lim_{\beta_i \to \infty} \tilde{\lambda} = \tilde{k} \)
- As \( \beta_i \to \infty \):

\[
\sum_{i=1}^{m} \Lambda_i = \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}^* = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{\lambda}_{ij}
\]

- \( PoA(\beta) < \infty \)
$\text{PoA}(\Lambda)$

$\beta = 40$
$\beta = 70$
$\beta = 100$
Price of Anarchy in Public Services *EJORS*, 2013.
What about the controllers?
What about the controllers?

Mathematical modelling of patient flows to predict critical care capacity required following the merger of two District General Hospitals into one.
Mathematical modelling of patient flows to predict critical care capacity required following the merger of two District General Hospitals into one. *Anaesthesia*
Divert?

RG

NH

Divert?
\[(K_{\text{NH}}, K_{\text{RG}})\]
\[ A = \begin{pmatrix}
(U_{NH}(1,1) - t)^2 & \cdots & (U_{NH}(1, c_{RG}) - t)^2 \\
(U_{NH}(2,1) - t)^2 & \cdots & (U_{NH}(2, c_{RG}) - t)^2 \\
\vdots & \ddots & \vdots \\
(U_{NH}(c_{NH},1) - t)^2 & \cdots & (U_{NH}(c_{NH}, c_{RG}) - t)^2
\end{pmatrix} \]

\[ B = \begin{pmatrix}
(U_{RG}(1,1) - t)^2 & \cdots & (U_{RG}(1, c_{RG}) - t)^2 \\
(U_{RG}(2,1) - t)^2 & \cdots & (U_{RG}(2, c_{RG}) - t)^2 \\
\vdots & \ddots & \vdots \\
(U_{RG}(c_{RG},1) - t)^2 & \cdots & (U_{RG}(c_{RG}, c_{RG}) - t)^2
\end{pmatrix} \]
Theorem.
Let $f_h(k) : [1, c_{\tilde{h}}] \rightarrow [1, c_h]$ be the best response of player $h \in \{NH, RG\}$ to the diversion threshold of $\tilde{h} \neq h$ ($\tilde{h} \in \{NH, RG\}$). If $f_h(k)$ is a non-decreasing function in $k$ then the game has at least one Nash Equilibrium in Pure Strategies.
$\text{argmin}_t (\text{PoA}(x))$
Measuring the Price of Anarchy in Critical Care Unit Interactions, *Journal of Operational Research*
$C^{(1)} - c^{(1)}$

\[
\begin{array}{c}
\mu_2^{(1)} \\
1 - p^{(1)} \\
\mu_1^{(1)} \\
\Lambda \\
\mu_1^{(2)} \\
1 - p^{(2)} \\
\mu_2^{(2)} \\
C^{(2)} \\
C^{(1)} - c^{(1)}
\end{array}
\]
$C = (4, 4), \quad N = (3, 5), \quad p = (0.2, 0.8), \quad \mu_1 = (1.0, 1.0), \quad \mu_2 = (0.2, 0.2)$
arrival

? ? ?

service
Generate population for all sources

while $clock < simulation\ time$ do
    Randomly group agents from each source
    Run a simulation of process
    Identify worst performing agents
    Reproduce agents with mutation
end