\[ \begin{pmatrix} (2, 2) & (5, 0) \\ (0, 5) & (4, 4) \end{pmatrix} \]
\( k = 1 \)
\( \mathcal{P}_1 = \{1, 2\} \)
\( c_1 = 1 \) and \( c_2 = x \)
\( r = 1 \)

The Nash flow minimises:

\[
\Phi(y, 1 - y) = \sum_{e=1}^{2} \int_{0}^{f_e} c_e(x) \, dx = \int_{0}^{y} 1 \, dx + \int_{0}^{1-y} x \, dx
\]

\[
= y + \frac{(1 - y)^2}{2} = \frac{1}{2} + \frac{y^2}{2}
\]

\( \Rightarrow \tilde{f} = (0, 1) \)
- $k = m$
- $|\mathcal{P}_i| = n + 1$
- $r_i = \Lambda_i$
- \( k = m \)
- \(|\mathcal{P}_i| = n + 1\)
- \( r_i = \Lambda_i \)
Theorem: Assuming $\sum_{i=1}^{m} \Lambda_i < \sum_{j=1}^{n} c_j \mu_j$ we have:

$$\lim_{\beta_i \to \infty} \text{PoA}(\beta) < \infty \text{ for all } i \in [m]$$

The price of anarchy increases with worth of service, up to a point.

Proof.

- $\lim_{\beta_i \to \infty} \lambda^* = k^*$ and $\lim_{\beta_i \to \infty} \tilde{\lambda} = \tilde{k}$
- As $\beta_i \to \infty$:

  $$\sum_{i=1}^{m} \Lambda_i = \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}^* = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{\lambda}_{ij}$$

- $\text{PoA}(\beta) < \infty$
Price of Anarchy in Public Services *EJORS*, 2013.
What about the controllers?
What about the controllers?

Mathematical modelling of patient flow to predict critical care capacity required following the merger of two District General Hospitals into one.

Submitted to Anaesthesia
Mathematical modelling of patient flows to predict critical care capacity required following the merger of two District General Hospitals into one., Submitted to Anaesthesia
Divert?

RG

NH

Divert?
\[ K_{NH} \quad K_{RG} \]

\[ \lambda_h^{(a)} \quad \lambda_h^{(c)} \]

\[ \lambda_h^{(b)} \quad \lambda_h^{(d)} \]
\[ A = \begin{pmatrix} (U_{NH}(1, 1) - t)^2 & \ldots & (U_{NH}(1, c_{RG}) - t)^2 \\ (U_{NH}(2, 1) - t)^2 & \ldots & (U_{NH}(2, c_{RG}) - t)^2 \\ \vdots & \ddots & \vdots \\ (U_{NH}(c_{NH}, 1) - t)^2 & \ldots & (U_{NH}(c_{NH}, c_{RG}) - t)^2 \end{pmatrix} \]

\[ B = \begin{pmatrix} (U_{RG}(1, 1) - t)^2 & \ldots & (U_{RG}(1, c_{RG}) - t)^2 \\ (U_{RG}(2, 1) - t)^2 & \ldots & (U_{RG}(2, c_{RG}) - t)^2 \\ \vdots & \ddots & \vdots \\ (U_{RG}(c_{RG}, 1) - t)^2 & \ldots & (U_{RG}(c_{RG}, c_{RG}) - t)^2 \end{pmatrix} \]
Theorem.
Let \( f_h(k) : [1, c_h] \rightarrow [1, c_h] \) be the best response of player \( h \in \{\text{NH, RG}\} \) to the diversion threshold of \( \bar{h} \neq h \ (\bar{h} \in \{\text{NH, RG}\}) \). If \( f_h(k) \) is a non-decreasing function in \( k \) then the game has at least one Nash Equilibrium in Pure Strategies.
argmin_t (PoA(x))
Measuring the Price of Anarchy in Critical Care Unit Interactions, Submitted to OMEGA
$C = (4, 4), \ N = (3, 5), \ p = (0.2, 0.8), \ \mu_1 = (1.0, 1.0), \ \mu_2 = (0.2, 0.2)$