Evolutionary Game Theory

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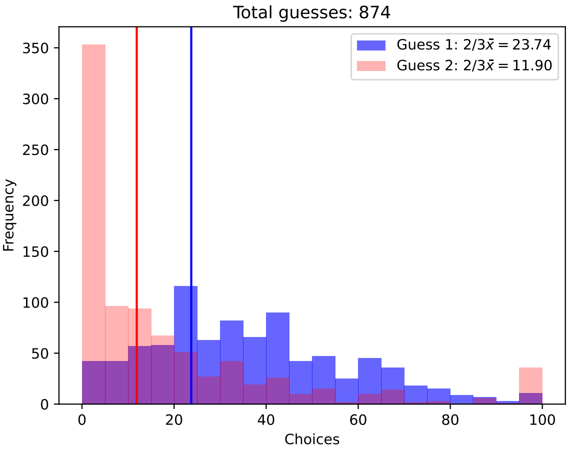
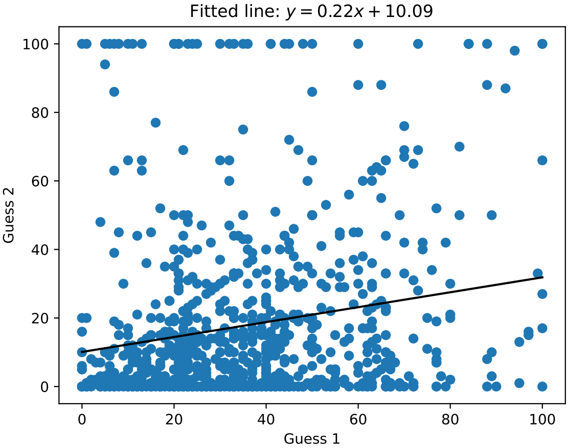
PRIFYSGOL CAERDYDD

Two Thirds of the Average

- Pick an integer between 0 and 100 (inclusive);
- Closest to two thirds of the average of all picked numbers wins.


Total guesses: 874
Guess 1: $2/3\bar{x} = 23.74$
Guess 2: $2/3\bar{x} = 11.90$
Frequency
Choices
0
50
100
150
200
250
300
350
0
20
40
60
80
100


Fitted line: $y = 0.22x + 10.09$
Guess 2
Guess 1
0
20
40
60
80
100

Definition

Considering an infinite population of individuals each of which represents an action from $\mathcal{A}$, we define the population profile as a vector $x \in [0,1]_{\mathbb{R}}^{|\mathcal{A}|}$. Note that:

$$\sum_{i\in\mathcal{A}} x_i = 1$$

Definition

The population dependent fitness of an individual of type i in a population x is denoted as $f_i : \mathbb{R}^{101}_{[0,1]} \to \mathbb{R}$.

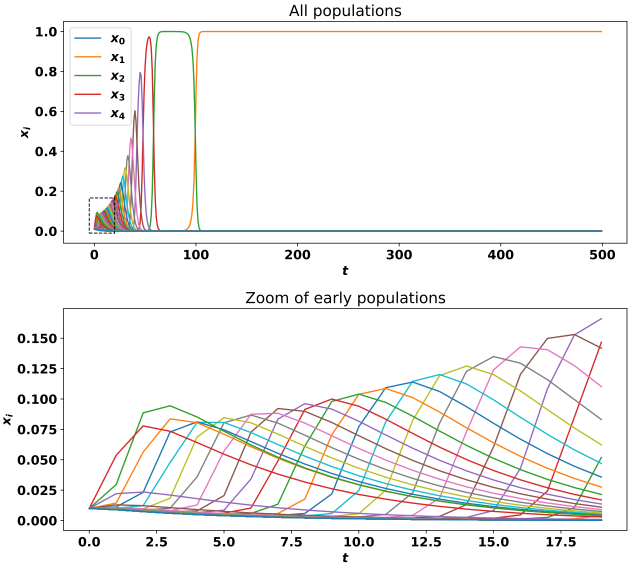
Definition

Replicator Dynamics Equation

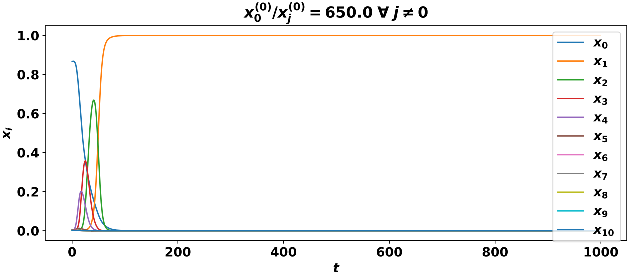
$$\frac{dx_i}{dt} = x_i(f_i(x) - \phi) \text{ for all } i$$

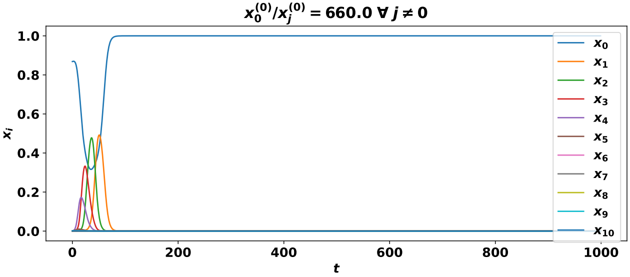
where:

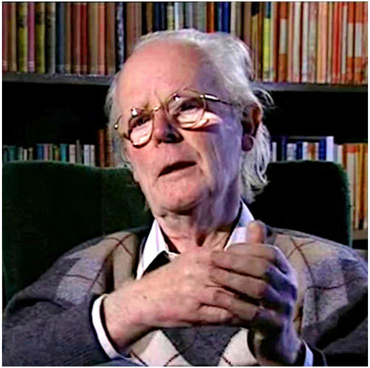
$$\phi = \sum_{i=0}^{N} x_i f_i(x)$$


All populations
x_0
x_1
x_2
x_3
x_4
x_i
t
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
Zoom of early populations
x_i
t
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

We see that over time, the population emerges to all guessing 1.
So everyone wins.
Note that everyone guessing 0 also is stable.


$x_0^{(0)}/x_j^{(0)} = 650.0\ \forall\ j \neq 0$
x_i
t
x_0
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
x_9
x_{10}


$x_0^{(0)}/x_j^{(0)} = 660.0\ \forall\ j \neq 0$
x_i
t
x_0
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
x_9
x_{10}



John Maynard-Smith[1] (1920 - 2004)

[1] J.M. Smith. *The Theory of Evolution*. A Pelican original. Penguin, 1977. ISBN: 9780140204339.

Definition

In a population game when considering a pairwise contest game we assume that individuals are randomly matched and play some game with utility matrices A, A^T. For a population profile x this gives a compact expression for the fitness:

$$f = Ax$$

Definition

In a pairwise interaction game the fitness of a strategy σ in a population x is given by:

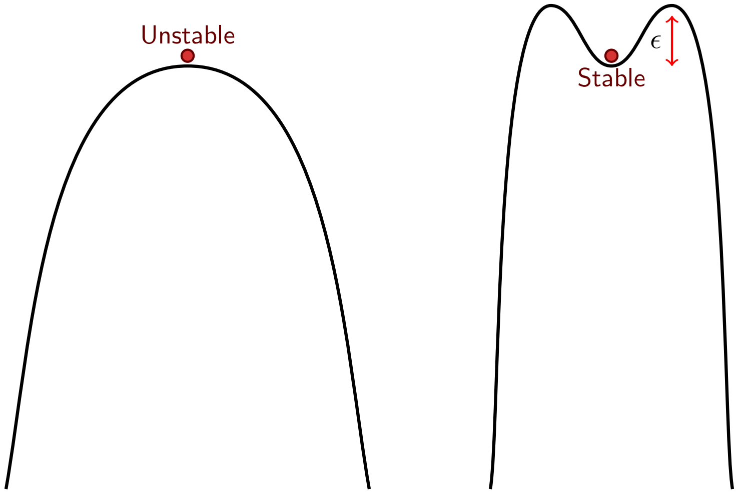
$$u(\sigma, x) = \sum_{i=1}^{|\mathcal{A}|} \sigma_i f_i(x)$$

Definition

A strategy σ^* is called an **Evolutionary Stable Strategy** if there exists an $0 < \bar{\epsilon} < 1$ such that for every $0 < \epsilon < \bar{\epsilon}$ and every $\sigma \neq \sigma^*$ σ^* is:

$$u(\sigma^*, x_\epsilon) > u(\sigma, x_\epsilon)$$

Where x_ϵ is the post entry population where a proportion ϵ of the population are σ.


Unstable
Stable
ϵ

Theorem

If σ^* is an ESS in a pairwise contest population game then for all $\sigma \neq \sigma^*$:
1. $u(\sigma^*, \sigma^*) > u(\sigma, \sigma^*)$ OR 2. $u(\sigma^*, \sigma^*) = u(\sigma, \sigma^*)$ and $u(\sigma^*, \sigma) > u(\sigma, \sigma)$
Conversely, if either (1) or (2) holds for all $\sigma \neq \sigma^*$ in a two player normal form game then σ^* is an ESS.

An evolutionary game theoretic model of rhino horn devaluation[a]

[a] Nikoleta E. Glynatsi, Vincent Knight, and Tamsin E. Lee. "An evolutionary game theoretic model of rhino horn devaluation". In: *Ecological Modelling* 389 (2018), pp. 33–40. ISSN: 0304-3800.

`nashpy.readthedocs.io`

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