Equilibria Behaviour of Rational Agents

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Auction

Dollar Auction

Top two bids pay.


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Definition

A N player normal form game consists of:

- A finite set of N players.
- Action set for the players: $\{\mathcal{A}_1, \mathcal{A}_2, \ldots \mathcal{A}_N\}$
- Payoff functions for the players: $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \cdots \times \mathcal{A}_N \rightarrow \mathbb{R}$

Example

Two friends must decide what movie to watch at the cinema. Alice would like to watch a sport movie and Bob would like to watch a comedy. Importantly, they would both rather spend their evening together than apart.

Definition

A strategy for a player with action set $\mathcal{A}$ is a probability distribution over elements of $\mathcal{A}$.
Typically a strategy is denoted by $\sigma \in [0,1]_{\mathbb{R}}^{|\mathcal{A}|}$ so that:

$$\sum_{i=1}^{\mathcal{A}} \sigma_i = 1$$

Definition

For a given strategy σ, the support of σ: $\mathcal{S}(\sigma)$ is the set of actions $i \in \mathcal{A}$ for which $\sigma_i > 0$.

Definition

Average payoff:

- $u_r(\sigma_r, \sigma_c) = \sigma_r A \sigma_c^T$
- $u_c(\sigma_r, \sigma_c) = \sigma_r B \sigma_c^T$

Definition

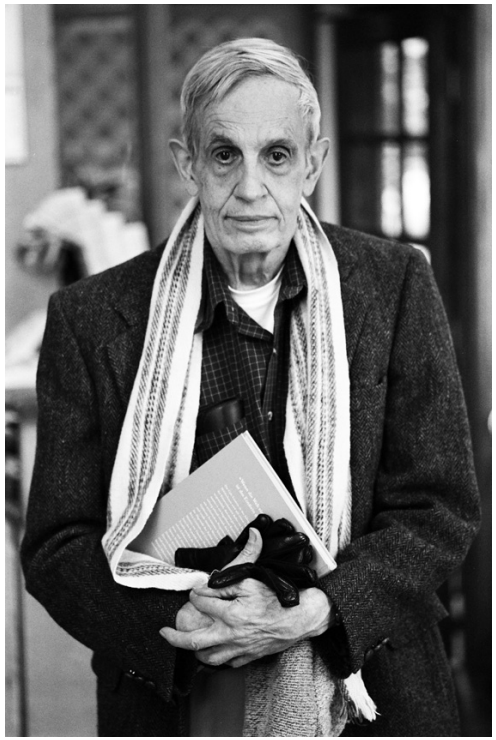
In a two player game $(A, B) \in \mathbb{R}^{m \times n^2}$ a strategy σ_r^* of the row player is a best response to a column players' strategy σ_c if and only if:

$$\sigma_r^* = \text{argmax}_{\sigma_r \in \mathcal{S}_1} \sigma_r A \sigma_c^T$$

Theorem

In a two player game $(A, B) \in \mathbb{R}^{m \times n^2}$ *a strategy* σ_r^* *of the row player is a best response to a column players' strategy* σ_c *if and only if:*

$$\sigma_{r^* i} > 0 \Rightarrow (A\sigma_c^T)_i = max_{k \in \mathcal{A}_2}(A\sigma_c^T)_k \text{ for all } i \in \mathcal{A}_1$$



John Nash[1] (1928 - 2015)

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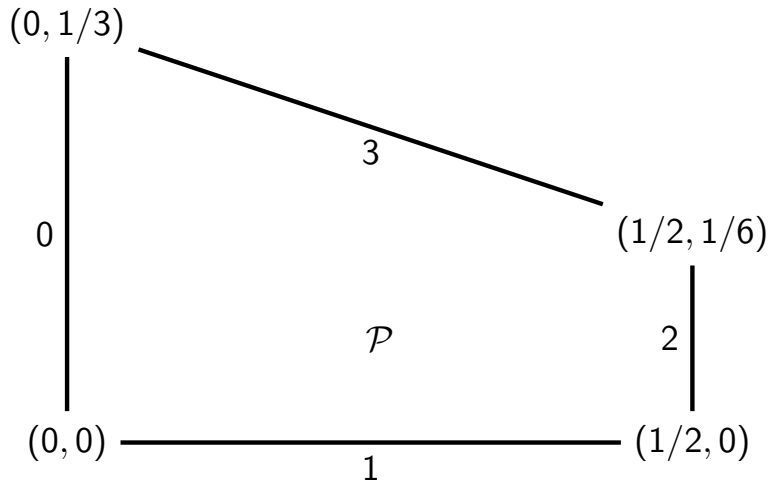
[1] John F. Nash. "Equilibrium points in *n*-person games". In: *Proceedings of the National Academy of Sciences* 36.1 (1950), pp. 48–49. DOI: 10.1073/pnas.36.1.48.

Definition

For a two player game $(A, B) \in \mathbb{R}_{>0}^{m \times n^2}$ the row/column player best response polytope $\mathcal{P}/\mathcal{Q}$ is defined by:

$$\mathcal{P} = \{x \in \mathbb{R}^m \mid x \geq 0; xB \leq 1\}$$

$$\mathcal{Q} = \{y \in \mathbb{R}^n \mid Ay \leq 1; y \geq 0\}$$


(0, 1/3)
3
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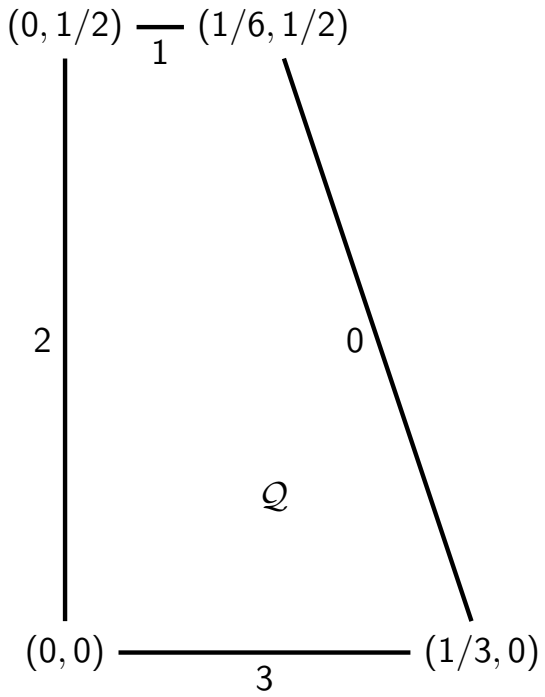
(1/2, 1/6)
$\mathcal{P}$
2
(0, 0)
1
(1/2, 0)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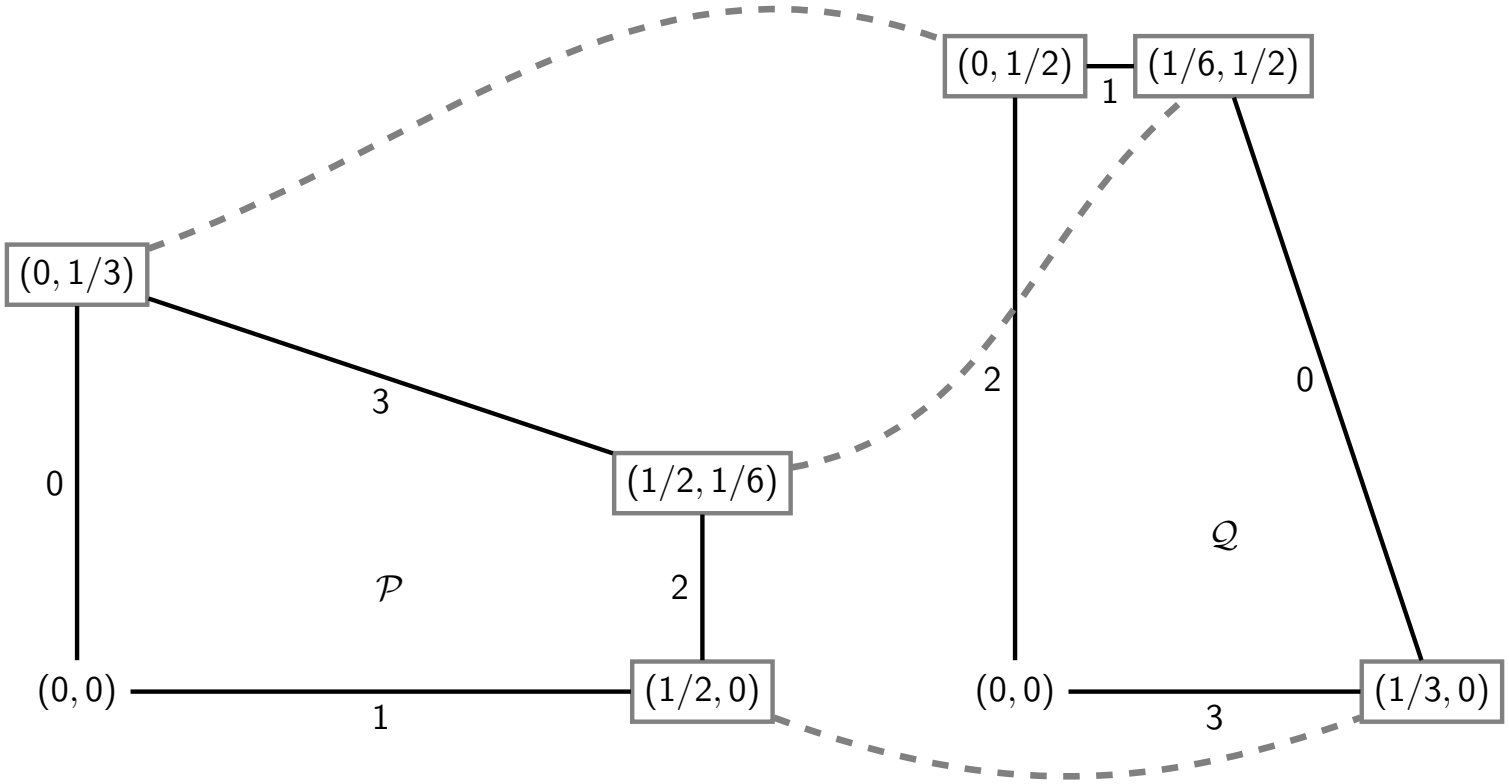
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1
3

L

emke-Howson Algorithm[a]

[a] C. E. Lemke and J. T. Howson Jr. "Equilibrium Points of Bimatrix Games". In: *Journal of the Society for Industrial and Applied Mathematics* 12.2 (1964), pp. 413–423. DOI: 10.1137/0112033.

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game theoretic model of the behavioural gaming that takes place at the EMS - ED interface[a]

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[a] Michalis Panayides, Vince Knight, and Paul Harper. "A game theoretic model of the behavioural gaming that takes place at the EMS - ED interface". In: *European Journal of Operational Research* 305.3 (2023), pp. 1236–1258. ISSN: 0377-2217. DOI: https://doi.org/10.1016/j.ejor.2022.07.001.

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