Routing games

1. A Nash flow is a flow in which ...
   (a) the total cost is minimised
   (b) every path carries equal flow
   (c) all traffic uses a single path
   (d) no individual can reduce their own cost by switching to another path
2. An optimal flow is the flow that ...
   (a) equals the Nash flow in every game
   (b) uses the most paths
   (c) no driver can improve on individually
   (d) minimises the total cost across all traffic
3. The price of anarchy measures ...
   (a) the number of paths in the network
   (b) the cost of the cheapest path
   (c) the ratio of the Nash flow cost to the optimal flow cost
   (d) how many drivers there are
4. Braess's paradox is the observation that ...
   (a) the Nash flow always equals the optimal flow
   (b) removing a road always helps
   (c) adding a new road can make every driver's commute worse
   (d) drivers never reach an equilibrium
5. A feasible flow that minimises the potential function Φ is ...
   (a) the optimal flow
   (b) a flow that uses no congested edges
   (c) never an equilibrium
   (d) a Nash flow
6. The optimal flow of a routing game equals the Nash flow of the game with ...
   (a) all costs set to zero
   (b) the costs doubled
   (c) each cost function replaced by its marginal cost
   (d) a single combined path

7. A feasible flow is one in which ...
   (a) the total cost is minimised
   (b) the flow on the paths of each commodity sums to its demand
   (c) no path is used
   (d) all flow uses a single path
8. Edge cost (latency) functions in a routing game are assumed to be ...
   (a) negative
   (b) constant
   (c) non-negative, continuous and non-decreasing in the flow
   (d) decreasing in the flow
9. The price of anarchy is always ...
   (a) at least 1
   (b) negative
   (c) exactly 1
   (d) at most 1
10. In Pigou's example (one fixed-cost route and one congestible route) the price of anarchy is ...
   (a) 1
   (b) 4/3
   (c) 8/7
   (d) 2
11. Braess's paradox arises because each driver ...
   (a) cannot find the shortest path
   (b) is forced onto a single route
   (c) pays a toll
   (d) ignores the extra congestion they impose on others (an externality)
12. At a Nash flow, all paths a commodity actually uses ...
   (a) are the cheapest in the whole network
   (b) carry equal flow
   (c) have different costs
   (d) have equal (and minimal) cost
13. The total cost of a flow is ...

(a) $\sum_e c_e(f_e) f_e$, summed over the edges

(b) the cost of the cheapest edge

(c) the price of anarchy

(d) the number of paths used

14. The optimal flow is found by ...

(a) maximising the total cost

(b) letting each driver choose selfishly

(c) using only one path

(d) minimising the total cost over all feasible flows

15. Marginal-cost pricing aligns the selfish and social outcomes because ...

(a) it sets all costs to zero

(b) it forbids the use of congested edges

(c) it removes all congestion

(d) it makes each driver internalise the congestion they cause

16. A routing game is written (G, r, c), where r denotes the ...

(a) number of edges in the network

(b) price of anarchy

(c) reciprocal of the cost

(d) amount of traffic (demand) each source-sink commodity must route

17. The potential function of a routing game is ...

(a) $\Phi(f) = \sum_e \int_0^{f_e} c_e(x)\, dx$

(b) $\Phi(f) = \sum_e f_e$

(c) $\Phi(f) = \max_e c_e(f_e)$

(d) $\Phi(f) = \sum_e c_e(f_e) f_e$

18. The marginal cost of an edge with cost function c is ...

(a) $c^*(x) = \int_0^x c(t)\, dt$

(b) $c^*(x) = c(x)/x$

(c) $c^*(x) = \frac{d}{dx}\big(x\, c(x)\big)$

(d) $c^*(x) = c'(x)$

19. The cost of a path P at flow x is ...

(a) the number of edges it contains

(b) the sum of the edge costs along it, $c_P(x) = \sum_{e \in P} c_e(x)$

(c) the cost of its cheapest edge

(d) the cost of its most expensive edge

20. In Braess's paradox, adding a zero-cost road changes the Nash flow so that its cost ...

    (a) rises, even though a new option was added

    (b) becomes zero

    (c) always falls

    (d) stays exactly the same