$$f(S_1) = (R_1, R_2, R_3)$$
$$f(S_2) = (R_2, R_3, R_1)$$
$$f(S_3) = (R_1, R_2, R_3)$$
$$g(R_1) = (S_2, S_1, S_3)$$
$$g(R_2) = (S_1, S_3, S_2)$$
$$g(R_3) = (S_1, S_2, S_3)$$

$$M(S_1) = R_1$$

Now consider S_2, R_2 is top if is unmatched.

$$M(S_1) = R_1$$
$$M(S_2) = R_2$$

Now consider S_3, R_1 is top of preference but it IS matched.

But R_1 prefers it's current matching.

So S_3 remains unmatched and we remove R_1 from its preference function

$$f(S_3) = (R_2, R_3)$$

Now consider S_3, top is R_2, R_2 is currently matched to S_2. But R_2 prefers S_3 to current matching:

$$M(S_1) = R_1$$
$$M(S_3) = R_2$$

Now consider S_2 with the updated preference function:

$$f(S_2) = (R_3, R_1)$$

The top preference for S_2 is R_3 and R_3 is unmatched.

$$\boxed{\begin{aligned} M(S_1) &= R_1 \\ M(S_2) &= R_3 \\ M(S_3) &= R_2 \end{aligned}}$$

$$f(S_1) = (R_4, R_1, R_2, R_3)$$
$$f(S_2) = (R_2, R_3, R_4, R_1)$$
$$f(S_3) = (R_1, R_4, R_2, R_3)$$
$$f(S_4) = (R_1, R_2, R_4, R_3)$$
$$g(R_1) = (S_4, S_2, S_1, S_3)$$
$$g(R_2) = (S_1, S_3, S_4, S_2)$$
$$g(R_3) = (S_1, S_4, S_2, S_3)$$
$$g(R_4) = (S_4, S_1, S_2, S_3)$$

Let us consider S_1, the top pref. is R_4:

$$M(S_1) = R_4$$

Let us consider S_2, the top pref is R_2:

$$M(S_1) = R_4$$
$$M(S_2) = R_2$$

Let us consider S_3, the top pref is R_1:

$$M(S_1) = R_4$$
$$M(S_2) = R_2$$
$$M(S_3) = R_1$$

Let us consider S_4, the top pref is R_1. R_1 is currently matched. The pref func of R_1 is

$$g(R_1) = (\underline{S_4}, S_2, S_1, \underline{S_3})$$

R_1 prefers S_4 so:

$$M(S_1) = R_4$$

$$M(S_2) = R_2$$

$$M(S_4) = R_1$$

Let us consider S_3 with updated preference fun:

$$f(S_3) = (R_4, R_2, R_3)$$

The top pref is R_4 which is currently matched to R_1

$$g(R_4) = (S_4, S_1, S_2, S_3)$$

S_3 remains unmatched and has updated pref. fn:

$$g(S_3) = (R_2, R_3)$$

Let us consider S_3, top pref is R_2. R_2 is matched to S_2

$$g(R_2) = (S_1, S_3, S_4, S_2)$$

keep repeating until all matched.

1. This is not a valid game: inconsistent actions in info set $\{d, e\}$

2. Same for info set $\{b, c\}$.

3. This $\underline{\underline{\text{is}}}$ a game.

- 2 subgames:

$\boxed{\begin{array}{l}\text{– at node } b. \\ \text{– at node } c\end{array}}$

- to derive NFG rep:

$$A_1 = \{ (\underset{a}{A}, \underset{\{d,e\}}{T}), (A,H), (B,T), (B,H) \}$$

$$A_2 = \{ (T,D), (T,C), (H,D), (H,C) \}$$

This gives

$$M_r = \begin{array}{c|cccc} & TD & TC & HD & HC \\ \hline AT & 1 & 1 & -1 & 1 \\ AH & -2 & -2 & 2 & 2 \\ BT & 20 & 10 & 20 & 10 \\ BH & 20 & 10 & 20 & 10 \end{array}$$

$$M_c = \begin{pmatrix} -1 & -1 & 1 & 1 \\ 2 & 2 & -2 & -2 \\ 20 & 10 & 20 & 10 \\ 20 & 10 & 20 & 10 \end{pmatrix}$$

this gives 4 pure equilibria:

$\{(BT, TD), (BT, HD), (BH, TD), (BH, HD)\}$

Looking at subgame at b with

$$A_1 = \{H, T\}$$

$$A_2 = \{H, T\}$$

$$M_r = \begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix} \quad M_c = \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix}$$

This has no pure N.E.
So the 4 previous N.E. are not subgame perfect.
Let us find the N.E. for this subgame:

Let $\sigma_1 = (x, 1-x)$

we have, from best response condition:

$$-2x + 1 - x = 2x - (1-x)$$

$$2 = 6x$$

$$x = \frac{1}{3}$$

N.E. $\rightarrow$ player 1 pick H $\approx \frac{1}{3}$ of the time.

Let $\sigma_2 = (y, 1-y)$

We have

$$2y - 2(1-y) = -y + 1 - y$$

$$6y = 3$$

$$y = \frac{1}{2}$$

NE → has player 2 play H $\frac{1}{2}$ at the time

Recalling the order:

$$A_1 = (AT, AH, BT, BH)$$

$$A_2 = (TD, TC, HD, HC)$$

So subgame perfect N. E.:

$$\left.\begin{aligned}\sigma_1 &= \left(0, 0, \frac{2}{3}, \frac{1}{3}\right)\\ \sigma_2 &= \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right)\end{aligned}\right\}$$