Tin Can Example - Writing Only

QUESTION:

Consider a cylindrical tin can with radius r and height h. Let its volume V be non-zero and fixed. Find the relationship between h and r such that the surface area of the tin can is minimised.

SOLUTION:

First, let r be the radius of the cylinder, h be its height, and V be its volume. We have expressions for the tin's volume and surface area:

As the volume V is fixed, then we have a relationship between r and h that is dependant on V:

Therefore the surface area now becomes a function of V and r:

The surface area is minimised when its derivative is equal to zero:

Then solving gives $\tilde{r}$, the value of r that minimises the surface area, in terms of V:

However, as mentioned above, the volume V is fixed, and is itself a function of r and h. Substituting this in gives an implicit relationship between $\tilde{r}$ and h:

Solving for either $\tilde{r}$ or h will give an explicit relationship between $\tilde{r}$ and h, as required.

Therefore, for a fixed non-zero volume cylinder, the relationship between the radius r and height h that minimised the surface area is $h = 2r$.