4. (a) Provide definitions for the following terms:

- Normal form game. [1]
- Strictly dominated strategy. [1]
- Weakly dominated strategy. [1]
- Best response strategy. [1]
- Nash equilibrium. [1]

For the remainder of this question consider the battle of the sexes game:

$$\begin{pmatrix} (1,-1) & (-2,2) \\ (-3,3) & (1,-1) \end{pmatrix}$$

(b) By clearly stating the techniques used: obtain all (if any) pure Nash equilibrium. [4]

(c) Plot the utilities to player 1 (the row player) assuming that the 2nd player (the column player) plays a mixed strategy: $\sigma_2 = (y, 1-y)$. [2]

(d) Plot the utilities to player 2 (the column player) assuming that the 1st player (the row player) plays a mixed strategy: $\sigma_1 = (x, 1-x)$. [2]

(e) Assuming that player 1 plays the mixed strategy $\sigma_1 = (x, 1-x)$, show that player 1's best response x^* to a mixed strategy $\sigma_2 = (y, 1-y)$ is given by:

$$x^* = \begin{cases} 0, & \text{if } y < 3/7 \\ 1, & \text{if } y > 3/7 \\ \text{indifferent}, & \text{otherwise} \end{cases}$$

Similarly show that player 2's best response y^* is given by:

$$y^* = \begin{cases} 0, & \text{if } x > 4/7 \\ 1, & \text{if } x < 4/7 \\ \text{indifferent}, & \text{otherwise} \end{cases}$$

[4]

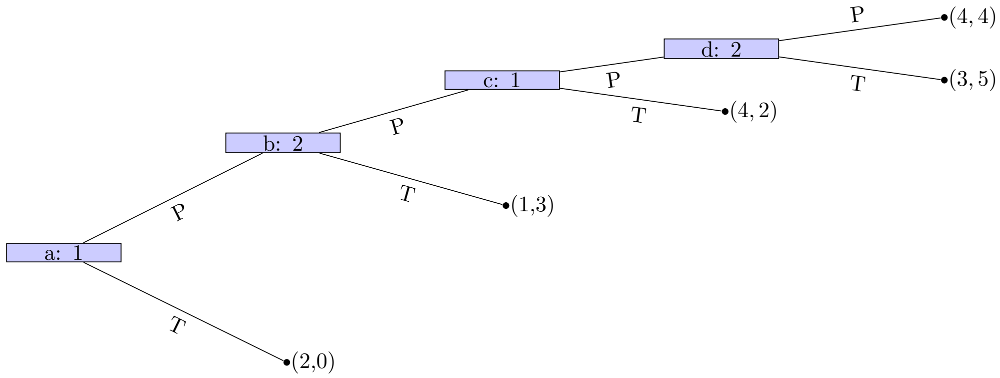
(f) Use the above to obtain all Nash equilibria for the game.

[2]

(g) Confirm this result by stating, proving and using the Equality of Payoffs theorem.

[6]

5. (a) Consider the centipede game shown below:



Obtain a subgame perfect Nash equilibrium for this game (you are expected to prove that it is a subgame perfect Nash equilibrium).

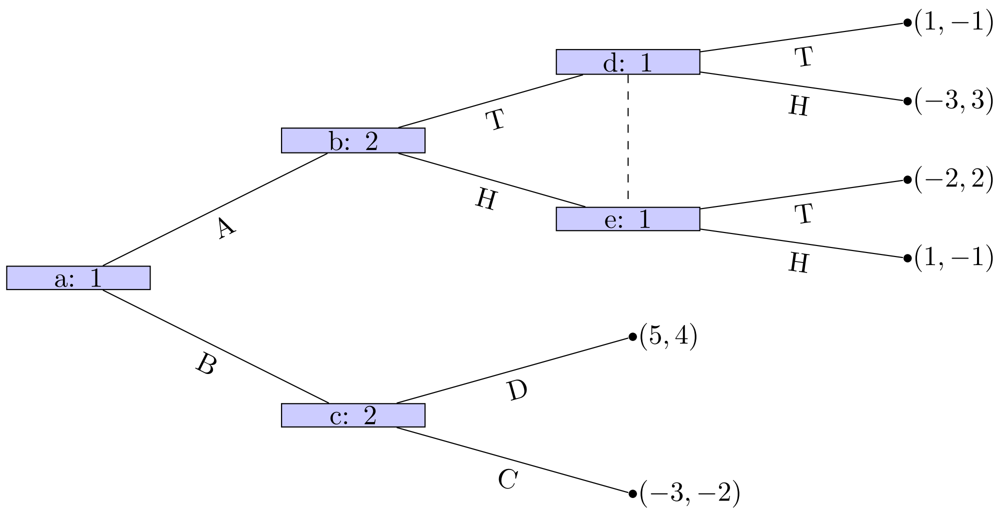
[11]

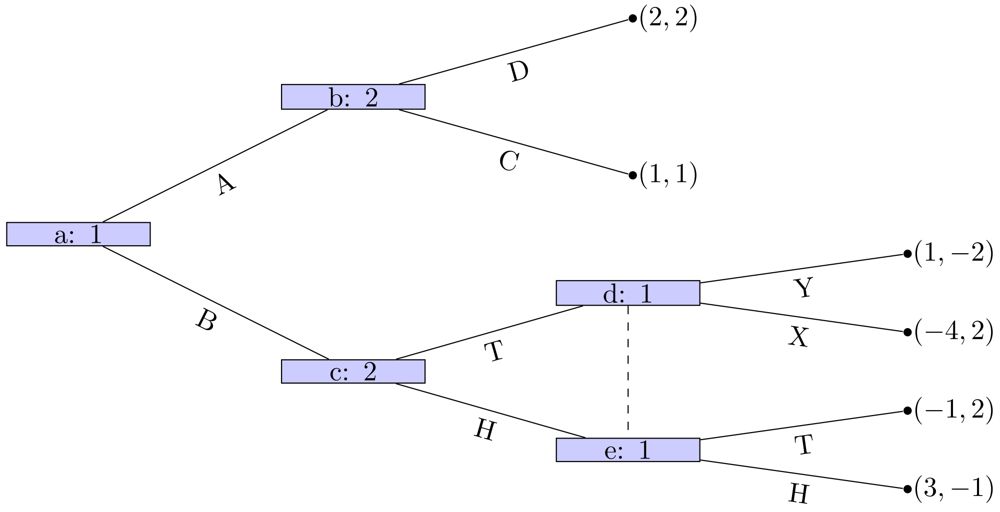
(b) Prove the following theorem:

"For any finitely repeated game, any sequence of stage Nash profiles gives the outcome of a subgame perfect Nash equilibrium."

[4]

(c) If they exist identify (prove) all subgame perfect Nash equilibrium for the following two games:





[10]

6. (a) Define a stochastic game.

[4]

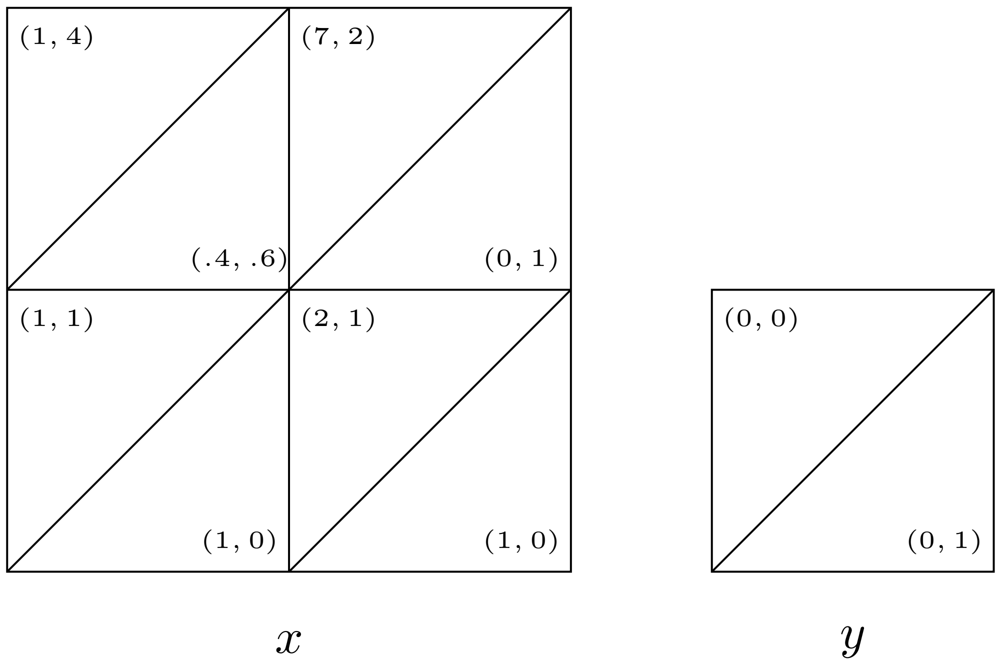
(b) Define a Markov strategy.

[2]

(c) Give the conditions for Nash equilibrium in a stochastic game.

[3]

(d) Obtain the pure strategy Nash equilibria (if it exists) for the following game with $\delta = .5$:



[16]