



Postgraduate Certificate in University Teaching and Learning

Module 1 Submission Cover Sheet

Participant number:

Participant name: Vincent Knight

School: Cardiff School of Mathematics

Mentor: Professor Paul Harper

This portfolio is submitted in fulfilment of the requirements for PCUTL: Module 1.

I declare that I have completed the compulsory elements of the programme as follows:

- a) workshops;
- b) 3 reflective journals;
- c) one mentor Peer Review of Learning and Teaching and accompanying resources;
- d) one annotated lesson plan with resources;
- e) Mapping of learning to date against the UKPSF.

I confirm that the evidence contained within this submission has been collected by me during the last 3 years, while teaching and/or supporting learning on recognised H.E. provision at Cardiff University

Unfair Practice: Plagiarism and Collusion

Plagiarism: In the University Academic Regulations Handbook, plagiarism is defined as 'using the words or ideas of others without acknowledging them as such and submitting them for assessment as though they were one's own work' (para 2.1.1). Plagiarism includes direct copying, close paraphrase, the unacknowledged use of ideas developed by others and commercial essay bank services.

Collusion: In the University Academic Regulations Handbook, collusion is said to occur when 'work that has been undertaken by or with others is submitted and passed off as solely the work of one person' (para 2.1.2). Where this is done with the knowledge of the originator, both parties can be considered to be at fault.

By submitting this portfolio, you are confirming that it is your own work and does not involve plagiarism or collusion.

The word count for the text of this submission is:

Signed:

Date:

Note: Portfolio assessors are entitled to reject any portfolio that does not have a signed copy of this form attached.



Postgraduate Certificate in University Teaching and Learning

Module 1: Summative assessment

To be completed by Participant

Name of participant:

School:

Verification of contents

Please state the location of the following evidence:

	Linked ILO	Location of compulsory evidence	Confirmed by marker
1) Three reflective journals: <ul style="list-style-type: none">- The context of UKHE and my role within it- Who am I? How do I learn?- So what for my lesson planning?	1+ 3	Journals 1 to 3.	
2) One PRLT with mentor and accompanying resources and reflection	4	PRLT Outputs.	
3) One annotated lesson plan with accompanying resources	2+3	Attached lesson plan and resources.	
4) Mapping of learning against the UKPSF		Attached mapping in PRLT output.	

Postgraduate Certificate in University Teaching and Learning

Summative Assessment of Reflective Teaching Portfolio: *To be completed by Participant*

Name of participant:

School / Directorate:

Self assessment of your achievement of the Module's Intended Learning Outcomes:

Learning Outcome	Location of evidence	Self-assessment including commentary to assessors
1. Describe the local and national contexts with respect to UK HE policy both generically and in their subject, and consider their role(s) within it.	I mainly address this in Journal 1 but refer to it in all my journals.	This was perhaps the ILO that I found the most troublesome. Nevertheless I would say that my journals demonstrate that the core ideas of the module have been understood and that the main purpose of the submission has been addressed.
2. Plan and run sessions that support student learning by giving active roles to students, fostering critical and independent thinking according to the standards of their subject.	My lesson plan and PRLT are evidence of this.	Based on my PRLT and the lesson plan I think I have demonstrated a sophisticated grasp of a variety of ideas that give active roles to students. I have addressed the standards of the subject in a comprehensive and imaginative way.
3. Explore the relationship between research, scholarship, related professional activities and teaching and learning as relevant to their own teaching practice.	This is mainly addressed in my journals.	I feel that I am at the beginning of the adventure when it comes to this ILO although my journal entries do address the main purpose of the submission.
4. Use PRLT to explore the impact of their teaching and/or support for learning on students' learning, and plan modifications accordingly.	My PRLT outputs, Journal 3 and my outlook to module 2 document.	I have discussed multiple modifications to my teaching as a result of my PRLT and furthermore have expressed other ways I plan to support students' learning in innovative way in my outlook to module 2. I feel that I have achieved this ILO fully.
5. Identify further professional development needs in relation to	This is addressed throughout my journals and my response to PRLT.	I have constantly discussed innovative and original ways in which I plan to further

teaching and/or supporting student learning.		my personal development.
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Evidencing the Programme Values:

Programme Value	Location of evidence	Self-assessment including commentary to assessors
1. An understanding of how students learn.	I demonstrate this in my lesson plan, PRLT and journals.	<p>I feel that I address this in my journals through a discussion of mathematical learning on the Kolb cycle. Note also that I address this (as noted by my mentor) in my lesson planning as I have various activities and methodologies that move students along the Kolb cycle. This is a value I feel that I have addressed well in a critical and creative fashion with personal insights into processes and outcomes. I can however improve on this by considering a variety of learning models as I continue my PCUTL journey.</p> <p>I feel I have addressed ILO 2,4 through this programme value.</p>
2. A commitment to reflection and evaluation and consequent improvement of professional practice.	Journals, PRLT response and outlook to module 2 document.	<p>The immediate evidence for this is by the fact that I'm doing PCUTL! I feel that the journals and discussions have very much helped me reflect on professional</p>

		<p>practice and I feel that I have evidenced a developing analytical approach to explaining my practice. As pointed out by my mentor in the PRLT there is room for this to be improved.</p> <p>I feel I have addressed ILO 2,3,4 and 5 through this programme value.</p>
<p>3. A respect for individual learners and for their development and empowerment, no matter what their circumstances.</p>	<p>Lesson plan and Journal 3.</p>	<p>I feel that I have addressed this issue quite well in particular at the end of Journal 3. I feel that comprehensive and detailed knowledge of the main module ideas have been demonstrated.</p> <p>I feel I have addressed ILO 1,2,3,4 and 5 through this programme value.</p>
<p>4. A commitment to scholarship in teaching, both generally and within their own discipline.</p>	<p>Journals and module 2 outlook document.</p>	<p>I feel that I have invested myself fully with quite personal reflections throughout my journals, which display my commitment to scholarship.</p> <p>I feel I have addressed ILO 3 through this programme value.</p>
<p>5. A commitment to the development of learning communities, including students, teachers and those engaged in learning support.</p>	<p>Journal 2 and 3.</p>	<p>This has been addressed through my journal in an analytical and logical manner.</p> <p>I feel I have addressed ILO 2,3 and 4 through this programme</p>

		value.
<p>6. A commitment to encouraging participation in higher education with respect to the issues of equality and diversity. In this regard, professional practice should be informed by equal opportunities legislation, policy and best practice.</p>	<p>Journal 3 and PRLT.</p>	<p>I gave specific consideration to this programme value in journal 3. Furthermore I evidenced original thinking through my PRLT.</p> <p>I feel I have addressed ILO 2,4 and 5 through this programme value.</p>

PCUTL activities	Areas of Activity					Core Knowledge and understanding of:					Professional Values				
<p style="text-align: center;">Module 1</p> <p style="text-align: center;">UKPSF mapping</p>	Design and plan learning activities and / or programmes of study	✓				Teach and / or support learning	✓			✓	Assess and give feedback to learners				
	Develop effective learning environments and approaches to student support and guidance			✓		Engage in continuing professional development in subjects / disciplines and their pedagogy, incorporating research, scholarship and the evaluation of professional practices	✓				The subject material	✓			
	Appropriate methods for teaching and learning in the subject area and at the level of the academic programme	✓		✓	✓	How students learn, both generally and in their subject /disciplinary area(s)	✓				The use and value of appropriate learning technologies	✓	✓		
	Methods for evaluating the effectiveness of teaching			✓	✓	The implications of QA and QE for academic and professional practice with a particular focus on teaching	✓				Respect for individual learners and diverse learning communities	✓	✓		
	The implications of QA and QE for academic and professional practice with a particular focus on teaching	✓				Respect for individual learners and diverse learning communities	✓	✓			Promote participation in higher education and equality of opportunity for learners	✓	✓		
Respect for individual learners and diverse learning communities			✓		Use evidence-informed approaches and the outcomes from research, scholarship and CPD				✓	Acknowledge the wider context in which HE operates recognising the implications for professional practice.	✓				
Use evidence-informed approaches and the outcomes from research, scholarship and CPD					Acknowledge the wider context in which HE operates recognising the implications for professional practice.	✓									
<p style="text-align: center;">Other experiences illustrated in appendices and reflections</p>															
Written text															

PCUTL - Module 1: Covering Claim

Vincent Knight

October 25, 2012

The ILOs for this journal are:

- Describe the local and national contexts with respect to UK HE policy, both generically and in their subject, and consider their role(s) within it.
- Plan and run sessions that support student learning by giving active roles to students, fostering critical and independent thinking according to the standards of their subjects.
- Explore the relationship between research, scholarship, related professional activities and teaching and learning as relevant to their own teaching practice.
- Use PRLT to explore the impact of their teaching and/or support for learning on students' learning, and plan modifications accordingly.
- Identify further professional development needs in relation to teaching and/or supporting student learning.

This portfolio contains a variety of materials. Firstly an annotated lesson plan is presented. This lesson plan was for a particular lesson that I enjoy giving and which lends itself well to giving active roles to students. My mentor has given a detailed review of this plan following which I have responded with ideas for how I am going to improve my teaching. Due to the timing of this lesson, I completed it before my reflective journals which I feel allowed me to reflect on the type of teacher I would like to be with relation to the discussion that occurred following the peer review. Before the appendices that include various teaching materials (as well as a page with detailed links to online materials) I include a short document that summarises further reflection that occurred as a result of discussions with my mentor. The role for that document is to give an outlook of how this portfolio will/can be carried on to Module 2.

In my first reflective journal I feel that I have addressed ILO 1 as well as ILO 3. The third ILO was further addressed in detail in the other reflective journals where I consider the various ways in which I learn, what I consider a teaching to be and finally what this implies for my teaching.

ILOs 2 and 4 are naturally addressed by the PRLT. As should be evident in my response to the peer review I have specifically explained how I plan to make certain modifications to my teaching.

Finally ILO 5 is addressed by the various discussions had with my mentor and PCUTL staff. I will concentrate on my capacity to mainstream the certain teaching methodologies as well as exploring other teaching models.

Through the various reflections that form that major part of this module (be it through the reflective journals or through the mentor peer review) it has become clear to me that I think of good practice in teaching as being able to “create learning opportunities”. This is a theme that I plan to emphasise and develop further as I progress through the various PCUTL modules. I hope to improve my own teaching of mathematics so as to ensure that I always provide students with the best learning opportunities that I am able to. Throughout PCUTL I hope to further understand a variety of learning and teaching modules as well as ensure that I am aware of a variety of technological solutions that ensure I give active roles to students. One way I plan to do this is to evaluate how students have responded to the various opportunities I presented them during my teaching this term (Autumn 2012).

VK - Game Theory Annotated Lesson Plan

1 Description

This is a lesson plan for the afternoon “teaching period” which aims to cover the Game Theory part of the syllabus of the MAT001: OR Methods MSc module.

The technical content of the module aims to cover the following topics:

- Normal Form Games
- Pure Nash Equilibrium
- Mixed Nash Equilibrium

The intended learning outcomes for this module.

On completion of the module a student should be able to:

- A Describe a general appreciation of the ideas of game theory.
- B Interpret the normal form of a game.
- C Use ideas such as common knowledge of rationality and dominance to identify dominated strategies.
- D Identify best responses to certain strategies in games.
- E Use the two above skills to identify pure Nash equilibria in games.
- F Describe the basic ideas of mixed strategies.
- G Compute mixed strategy equilibria using the equality of payoffs theorem.
- H Describe an appreciation of the relationship of state of the art research and the topics they have learnt.

2 Lesson plan

Time	ILO	Teacher Activity	Learner Activity	Resources
Before the class	A,H	Invite student to look at videos: <ul style="list-style-type: none"> • EU-EMS Interface • Introduction to mixed strategies 	View videos	Videos
0-20mins	A,C, D,E	Explain 2/3rds of average game	Play 2/3rds of average game	Slides + Handouts
20-40mins	B,C, D,E	Lecture on Normal Form Games and Pure Strategies	Listen	Slides + Videos
40-50mins	NA	Break	Break	NA
50-80mins	A,D, E	Run PD tournament	Active participation in PD tournament: <ul style="list-style-type: none"> • Group discussion of strategies • Inter group discussion of strategies • Duel 	Playing Cards
80-100mins	F,G	Lecture on Mixed Strategies	Listen	Slides + Sage Interact
100-130mins	F,G	Help if needed	In groups of 2 work on Sage lab sheet	Sage lab sheet and interact

3 Assessment

The above ILO would be assessed as part of the wider assessment for MAT001: part of the exam. This type of assessment is well suited to the subject area as it will allow for evaluation of each ILO.

4 Comments

This lesson plan is designed in such a way as to ensure that most learning styles will be catered for. Indeed students are initially introduced to Game Theory through a game itself. This should allow students to be put in the place of a decision maker and thus allow for the more complex concepts to be easier to understand. Furthermore, it is hoped that this will relax students and make way for better students participation throughout the course.

To complement the two participation activities there are two breaks scheduled which are put in place to ensure that students won't be tired and their concentration will not falter. Apart from these activities information will also be delivered through two short lectures and there is also a group exercise scheduled which will hopefully invite peer learning. This exercise is intentionally different to all other learning exercises presented in this period. It makes use of programming and relates to a video that students will hopefully watch before the lecture.

Finally there is a tutorial session scheduled that will allow for students to carry out classical pen and paper exercises which should help them prepare for the exam. This tutorial will be taken by 2 postgraduate students to hopefully offer a different perspective however I will still be present in case I am needed.

Output 1: PRLT – preparatory reflection

Name: Vincent Knight

Module Title: OR Methods

Level/Year of Study: MSc

Focus of PRLT: A Lecture on Game Theory

Number of Students present: 31

What are the aims of the session?

What I intend to do, how it relates to the rest of the course. Are there opportunities in this session / learning activity to explore any of CU's key strategic themes / explicitly consider employability issues etc?

I will be teaching game theory using a combination of methods presented through a classic lecture interrupted by various activities that require student participation. This session will ensure students gain understanding of game theoretical concepts through all stages of the Kolb Cycle. Issues relevant to employability are touched upon as students need to negotiate amongst themselves so as play certain role playing games.

What are the learning outcomes for this session? How am I going to help the students achieve them? Are the learning opportunities I'm creating inclusive?

See attached lesson plan.

Output 2: Dialogue summary / feedback on PRLT activity

Name: Paul Harper

Module Title: OR Methods

Level/Year of study: MSc

Focus of PRLT: A Lecture on Game Theory

Date: 02/10/2012

Session summary:

This session was on the topic of Game Theory, and was taught over a one half-day (3.5 hours) session. Vince had already been teaching to the same cohort all morning (albeit on a different topic) so it was an intensive day for him (and them)! Also one might expect a dip in energy levels and attentiveness immediately after lunch. However Vince captured the student's attention fully and kept their engagement in a manner that was a pleasure to observe. He should be applauded for the content and range of teaching methods and resources employed to suit different learning styles amongst the student cohort. The stated intended learning outcomes (ILOs) were completely met.

Clarity of outcomes:

Clear overview of session initially provided.

Planning and organisation:

It was evident that a huge amount of effort had gone into planning the entire session. An impressive range of learning resources and methods were used, including traditional lecture notes (slides), use of whiteboard, provision of handouts to students, showing videos, role-play games, computer labs etc. Furthermore Vince had given thought to the planning of tutorial assistance both during the lecture and in the computer-labs/tutorial sessions.

Methods/approaches:

The session was well introduced. Got students immediately to think for themselves by asking 'what is a game?' The different methods employed emphasised the key points to help meet the ILOs. The different elements of the session linked back neatly to these ILOs.

Delivery and pace:

Maintained a good pace. Dealt appropriately with students arriving late so as not to cause disruption Audible and clear voice. Good eye-contact. Slides were entirely suitable and readable. Handouts helped. Good rapport with students. Humour on occasions (especially early on) which actually worked well. Breaks were well timed. Recapped nicely at end of lecture session. Offered students the possibility of a tutorial session after the

computer lab but suggested they instead reflect on the day and work on the exercise sheet in their own time.

Content:

Entirely suitable for the level and introductory material to the subject. Numerical examples supported the theoretical workings. Computer session reinforced the lecture materials.

Student participation:

Ample opportunity was provided (e.g. by playing actual games; inviting responses from the students etc). Vince was interrupted on numerous for clarification, but dealt with this well and responded in an appropriate helpful manner i.e. one which reinforced the material but without sounding condescending. Actually having the students so evidently engaging and feeling comfortable asking questions demonstrated an environment entirely appropriate for an M-level course but one which can be very tricky to encourage in a Mathematics-based degree. (particularly noteworthy as this was day 1!). Clearly the Prisoner's Dilemma tournament and $\frac{2}{3}$ game were good fun and well received by the students, and reinforced the learning but in a 'hands-on' way. Asked students to work in pairs in the computer lab; partly because of potential problems with the server but also as a way to learn together - neat.

Use of learning resources:

Simply stated, superb! In fact I would recommend the majority of other lecturers in the School (including those far more experienced) come observe and learn from Vince about e-learning technologies. He demonstrates a thorough understanding of a range of appropriate and up-to-date resources and technologies, both for use within the session and for the students outside the classroom (such as creating video resources, on-line tools using Sage, materials available on personal website, use of Google+ for pre-session information etc.)

Reviewer's comments and suggestions

Commitment to the Professional Values of the UKHE:

1. An understanding of how students learn.
 - a. Varied type of learning materials that moved students through the Kolb Cycle.
 - b. VK can expand on this and explore further learning models.
2. A commitment to reflection and evaluation and consequent improvement of professional practice.
 - a. Spent a while planning.
 - b. Various discussions with colleagues and others.
 - c. VK can continue to expand on this.
3. A respect for individual learners and for their development and empowerment, no matter what their circumstances.

- a. Welcome questions.
 - b. Use of open source resources.
 - c. Multiple stages of the Kolb cycle.
 - d. Videos that catered to non English speakers.
 - e. VK can further explore different learning and styles to ensure he caters to them.
4. A commitment to scholarship in teaching, both generally and within their own discipline.
 - a. Doing PCUTL.
 - b. Has read some literature but VK can build a further knowledge of the education literature.
 5. A commitment to the development of learning communities, including students, teachers and those engaged in learning support.
 - a. Interaction with other member of staff.
 - b. Sharing of resources.
 - c. VK will benefit from further PCUTL module where he'll share teaching and learning techniques with other lecturers from CU.
 6. A commitment to encouraging participation in higher education with respect to the issues of equality and diversity. In this regard, professional practice should be informed by equal opportunities legislation, policy and best practice.
 - a. Use of notes that can be read on any system.
 - b. Videos that cater to non English speakers.
 - c. VK can explore further ways in which his notes can be made adaptable to a wider audience (further use of html).
 7. Use evidence-informed approaches and the outcomes from research, scholarship and CPD.
 - a. VK can certainly evidence his approaches, he will benefit from the rest of PCUTL to justify his methodologies.
 8. Acknowledge the wider context in which HE operates recognising the implications for professional practice.
 - a. VK has demonstrated this on multiple occasions through his journals and discussion.
 - b. After discussions it is apparent that VK is keen to further explore this.

Overall thoughts/considerations/reflections for Vince:

You worked really hard to deliver an outstanding session, enabling students to meet the ILOs whilst recognising different learning styles. The thoroughness in your planning paid off. You employed a range of resources that worked well for the format of the session (4-hours) as well as benefitting the students outside the classroom (case-studies, exercises, videos etc) Student's were kept engaged and at the same time challenged by an appropriate content. Your enthusiasm was tangible. You did run over time (against your lesson plan) but this didn't matter in the slightest given the flexibility in the day's teaching and overall module. It was entirely appropriate to stay with the plan (and in fact you added a break and spent time reinforcing some material) rather than speed-up, especially as the room was getting stuffy and students were getting tired. I spoke to several

students at the end of the day, and no one had a bad word to say about you. They really enjoyed the range of resources and came away from the day enthused. Congratulations!

Some areas for potential improvement/reflection:

Context: there was no mention at all in the session about the wider application of game theory. For example why is it taught within the module/programme and how is game theory used in practice and by whom?

A couple of errors in the slides crept in (spotted by students) - a couple of sighs from the back (mostly because they were initially struggling to understand the equations and the error didn't help) - of course it is tricky to have completely error-free slides but worth double checking slides beforehand.

Consider use of skeletal notes for handouts: just a personal preference, but I find use of skeletal notes with gaps in notes at important points stresses to students main results and helps with concentration. In your case though you used the whiteboard extensively to expand on the notes, but often does no harm for them to write something (from where I sat I actually observed (surprisingly) very few people writing anything...some were highlighting key equations but it might have done no harm for them have written them out for the first time themselves)

Be careful if appearing to 'pick-on' a student(s), even if you know them well and anticipate they won't mind. You only did this twice early on, and I appreciate for humour, but might make them feel awkward and moreover others in the class wondering why you have picked them out and yet no-one else.

Signed: _____
Capacity of reviewer: Mentor

Date: _____

Output 3: Reviewer /Reviewee Post PRLT Reflection

Reviewee's response:

What have I learned about my teaching /learning support practice?

This peer review was quite positive and in particular highlighted certain aspect of my teaching that I should continue and in particular try and to stream across the majority of my teaching:

- Use of learning resources.
- Pace of delivery.
- Student participation

There are however certain aspects that I must improve. In particular I will try and emphasise further the context within which the subject sits. After the class I had shared state of the art research materials on the topic of Game Theory with students. The hope of this was to ensure that students would understand the applicability of the work and how it fitted in to the MSc course. After discussions with my mentor however I don't feel that I emphasised this enough.

Other aspects include errors in my notes. This is certainly something that I need to endeavour to remove from my teaching.

With regards to the use of skeletal notes. I have given this some thought and feel that I will continue to use distribute "full" notes to the students. At the beginning of my teaching I emphasise that I want the students to "take responsibility" for their learning and as such I ensure that my entire set of teaching resources is available to them before every lecture. The fact that few students were writing is however something that needs to be addressed.

Finally with respect to the single student whose interaction with could have been understood as "picking on". This particular student is quite a popular student and I happen to know him quite well. When I was a PhD student he would often come and ask me for help and he did his final year project under my supervision. As such I have perhaps gained a little bit too much familiarity with him and teased him slightly during the lecture. Interestingly, I myself thought that this was inappropriate almost immediately. After the lecture I had a brief chat with the student in particular who assured me that he did not feel at all aggrieved and had in fact thought it was funny. Despite that, I completely agree with my Mentor and I should not tease students in any situation.

After some further discussion with my Mentor, we discussed that despite some of the students saying they had enjoyed my lesson this was of course a small sample. I will attempt to collect some general feedback from the students to see the overall consensus.

What I may do differently, think about next time? Is there anyone / a resource I can work with on this?

There are three aspects that I will modify in future teaching. Firstly I need to emphasise the context of the particular subject at the beginning of the teaching session. I will be sure to spend some time doing this and place the subject in the context of the module and the MSc program in general. It will be helpful to have the MSc program documents to base this on (these have been given to me).

I will continue to ensure that typos and mistakes are removed from my notes. I will also be sure to not give the appearance of “picking on” any student.

What would be a useful focus for my next PRLT?

(Think perhaps both about using PRLT to develop ideas raised above further, and also as a resource to support your evidencing of the PCUTL ILOs and elements of the UKPSF)

The main aspect that should be focused on in my next PRLT are to ensure that the positive aspects from this PRLT are transferred to other subjects. The particular subject area can be taught in an interactive and dynamic way. It would be beneficial to ensure that my next PRLT is for a different type of subject and in particular evaluate that I am able to mainstream my methodologies.

Signed (Participant) _____

Date: _____

Mentor sign off:

Module 1:

- **I confirm that we have framed our discussion in the context of discipline-specific pedagogy and the PVs outlined in Output 4.**
- **I also confirm that the participant has satisfactorily achieved Module 1 ILO4**
(Use PRLT to explore the impact of their teaching and/or support for learning on students' learning, and plan modifications accordingly.)

Signed (Mentor) _____

Date: _____

Output 4: Evidence that the ‘observed’ student-facing practice is explicitly underpinned by the Professional Values of PCUTL and UKHE. Please tick and / or annotate as appropriate.

	Evidence that PV is embedded in practice	Evidence of some awareness of relevance of PV to practice	Needs to be explored further. Specific questions to focus on could be....
1. An understanding of how students learn.		I have demonstrated this in my journals and lesson plan but also plan to further investigate learning models.	
2. A commitment to reflection and evaluation and consequent improvement of professional practice.		A large amount of time was spent planning but this can be further improved.	
3. A respect for individual learners and for their development and empowerment, no matter what their circumstances.		As noted by my mentor above I addressed this in multiple ways.	
4. A commitment to scholarship in teaching, both generally and within their own discipline.		I can build on this by gaining a larger knowledge of the relevant mathematical education literature.	
5. A commitment to the development of learning communities, including students, teachers and those engaged in learning support.		I actively discuss my teaching with other teachers on social networks.	

		As noted by my mentor I will benefit from the interactions I expect to have as I go through PCUTL.	
6. A commitment to encouraging participation in higher education with respect to the issues of equality and diversity. In this regard, professional practice should be informed by equal opportunities legislation, policy and best practice.		I cater for this in multiple ways as noted by my mentor. I look forward to further considering issues surrounding equality and diversity.	
7. Use evidence-informed approaches and the outcomes from research, scholarship and CPD.		I am aware of the need to evidence my approaches and will emphasise this aspect as I go through PCUTL further.	
8. Acknowledge the wider context in which HE operates recognising the implications for professional practice.		I have demonstrated this in my first journal and also plan on looking in to it more.	

UK Higher Education and My Role Within It

VK

The ILO for this journal:

- 1. Locate yourself in your discipline, your department, your University and wide national and international contexts.**
- 2. Explore the relationship between research, scholarship, related professional activities and teaching and learning as relevant to your own teaching practice.**

In this journal I will be describing my understanding of UK Higher Education as well as my place within it. I will also discuss the relationship between research, professional activities and my own teaching practice.

The Higher Education Funding Council for Wales use the funds distributed by the Welsh Government for Higher Education. In [3] it is stated that higher education must be provided in every region of Wales for the benefit of local learners and employers. Further aspects that are emphasised in such policy documents [3,4] are the importance of research led teaching, sustainability of universities, equal opportunities with widened access, and the importance of employability of graduates.

The aims and vision of Cardiff University are very much in line with the above [1] with particular current emphasis on building a vibrant postgraduate research community and an increase in international impact and outreach. To attract such students to Cardiff University the immediate question arises: "What makes studying mathematics at Cardiff University special?". The immediate answer is "nothing much". Cardiff is a capital city which has immediate advantages as well as political ramifications with regards to other Universities in Wales but I am choosing to avoid this topic. The physical building itself is not a nice one, mathematics departments in other universities are much situated in much nicer buildings. So perhaps the immediate answer is that there is nothing that special about studying mathematics at Cardiff. Upon a bit more reflection there is one huge attraction to studying mathematics in Cardiff: our Operational Research (OR) group. Operational Research is historically difficult to define but it is a branch of applied mathematics concerned with problem solving. The OR group at Cardiff is one of the biggest groups in the United Kingdom with experts in various applied fields such as Queueing Theory, Optimisation, Game Theory and Simulation. This positions us very strongly in the research community but is also translated into our teaching. Our undergraduate degree is one of the very few in the United Kingdom which offers specific modules in OR. Another big strength in our degree is our placement program. Every year we send out a growing number of students finishing their second year to work for a year in industry. These particularities obviously address the issue of employability of graduates but also place a certain level of responsibility on myself as a lecturer. I am on the front line of delivery for a number of these skills and must ensure that they are delivered correctly.

My personal research interests are in game theory and queueing theory. These two specialities belong to the field of Operational Research which can be broadly defined as the application of mathematical sciences to solving real world problems. The Operational Research group within the Cardiff School of Mathematics is one of the leading groups in the United Kingdom with a well developed international reputation for outstanding research. As part of this group I regularly attend international conferences and publish in leading journal ensuring I stay at the forefront of

my subject area.

My own teaching responsibilities are involved in four modules:

- OR Methods
- Advanced Statistical packages
- Game Theory
- Computer Science for Mathematics

The first two courses are postgraduate courses on the taught MSc program. The last two are in development and will be delivered for the first time during the 2013/2014 academic year. The Game Theory course will be taught to our final year students. The Computer Science course comes with a fundamental philosophical shift in the interpretation of what it is to be Mathematician. By teaching this to our first years we are stating that all graduates from our School will have a minimum level of programming skills. This is not common to many universities and again offers something unique to graduates of Cardiff University's School of Mathematics. I was particularly keen to get this module running and am keen to start preparing it. It will add to the growing employability profile of our graduates and also ensure to making mathematics relevant.

I consider myself lucky to have been given teaching responsibilities that are closely related to my research activities. This ensures that I will be able to deliver state of the art teaching at the forefront of internationally renowned research.

The particular subject areas that I teach are of particular interest to employers. Indeed Operational Research is well known to be a desirable degree by employers (due to the emphasis of applicability of the mathematics learned). Furthermore it also gives graduates a high level of learnability [5]. These aspects ensure my place within the goal of UK HE with respect to employability of graduates.

I am also involved in the supervision of multiple postgraduate research students. This is an aspect of teaching that I very much enjoy and hope to continue to develop. I also regularly produce and deliver teaching materials that are available on youtube. These short videos aid in the dispersion of Cardiff University's name on an international scale and further contribute to the goal of widening access. With my mentor Professor Harper, I also deliver regular outreach activities. This is something I particularly enjoy and hope to continue as a contribution to the university and country's strategies with regards to widening access and equal opportunities, I might revisit this in Module 2...

I very much enjoy my role and feel that I am strongly placed within the international, national and university education context.

Word Count: 909

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Who am I? How do I learn?

VK

The ILO for this journal:

- 1. Locate yourself in your discipline, your department, your University and wide national and international contexts.**
- 2. Explore the relationship between research, scholarship, related professional activities and teaching and learning as relevant to your own teaching practice.**

Answering the question of “Who am I?” in a 500 word journal seems like quite a challenging philosophical problem so I will most certainly omit certain details.

I have recently discovered that I fall into a category of individuals called “third culture kids” [5]. This is linked to the fact that I spent my childhood in a variety of countries and subsequently cultures (different to my parents’). I always consider myself extremely lucky to have had such an upbringing which exposed me to a variety of teaching styles, cultures and philosophies. For example I experienced corporal punishment. This is something that most people of my generation from the United Kingdom would not have experienced. On a less extreme note I did learn in a variety of different situations which I think is beneficial. Obviously this could also be understood to have a negative affect as I did not go through a homogeneous education system.

In an education context I would hope that the answer to the “Who am I?” question is that I’m an open minded individual eager to try and learn different styles of learning and teaching. This is of course based on a subjective self assessment and so I will now discuss results from a variety of tests that further enable me to identify the sort of learner and teacher that I am. I completely understand that these tests come with a health warning and should not be taken at face value. I anticipated to look at all test scores with a critical mindset (in all honesty I thought they wouldn't mean much at all but was pleasantly surprised when they seemed to either confirm certain preconception about myself I had and or when I thought about them a bit more I realised that there was perhaps something in them).

The first set of results is related to the Rezler test [6]. This test aims to reflect the environment in which I as a student like to learn. My scores reflect that I am much more “student structured” than “teacher structured”. I very much think this confirms what I would have suggested prior to the test. I’ve always liked to think of a teacher as someone who opens doors but as myself as the person going through them. This relates well to another test which aims to indicate my “Academic Locus of Control” [1]. This I understand to imply where I place responsibility for failure in learning. I score very highly on the “Internal” locus implying that I feel that I take responsibility for my learning. I again think that this seems to match what I recall from my student days. I’ve always blamed myself if I didn’t get a high enough score on an exam and never enjoyed getting into conversations with others blaming a Lecturer for not being good

enough. For example one of my favorite modules was one in which the Lecturer was perhaps not the best at conveying information and I had to go learn a lot by myself. The achievement of success felt so much better and I also believe to have learnt a lot more on that module than on any other. I suppose that taking this to the extreme I might ask: "why do I need a teacher?". I'm happy enough to answer that question immediately by referring to my earlier analogy: a teacher opens a door that a student must go through. I can perhaps also expand on that: sometimes the light goes out and the teacher needs to turn it back on and/or yell directions from the doorway.

Another test we undertook was the Assist test [8]. This test identifies learning styles and I seem to have a strategic style of learning (there seemed to have been an error with my scores and no information was given as to my proficiency for "depth" learning). This strategic style apparently points towards an "Intention to excel". This does seem to fit my personality, I would always tell my peers that unless they could guarantee 100% on an exam they should not leave early. I never left an exam early.

The final two tests are meant to be assessments on how I view teachers [2,9]. These all seem to confirm prior remarks relating to the fact that I seem to have an internal locus of control. A few interesting things appear however. I score quite highly as a motivator and carer. When I first read that my initial thoughts were simply that this test was obviously wrong as I don't have a reputation for "putting my arm around someones shoulder". Having thought a bit more I think it makes sense as I do care about the success and learning of students.

Finally it would seem that my trainer type is that of a Coach and a Director. Interestingly this places me on the Active Experimentation and Reflective Observation phases of the Kolb cycle. In particular to mathematics [3] this in turn implies that I am an Analyzer and a Synthesizer. This apparently says that I desire logical explanations and algorithms and am also capable of creating new individual ideas. This doesn't seem to alien but does imply that I am neither an Allegorizer or an Integrator which are students who like to consider new ideas by reformulating and/or comparing to known ones.

Another thing worth considering is the fact that I am a very visual learner. For example in the class my mentor reviewed me in I was using graphical software packages to show visualisations of theorems and I also used a fair bit of video. For mathematics I think this places me well as more often than not mathematics is taught in a rather "dry" manner and without much visualisation. In a sense for mathematics I think that it's "easy"/"more natural" to cater to none visual learners (you have to write mathematics and you have to do mathematics by design). So in a sense my tendency for "visual" teaching is probably a good thing as I will "by design" be anchored by the non visual aspects of mathematics (it is how I was taught). I also really enjoy role games and things like that which once again are not "classic" teaching methods so I think that is a good thing. Having said that another thing I suppose I need to consider is the fact that students of mathematics will have been taught in a classic way for a long time so perhaps they have "adapted their learning styles". For the course I'm currently teaching I have for two weeks now run little lab sessions that enable the students to further pursue ideas with a computer package. I guess this caters to the "Synthesizers" and visual learners. At the end of one of the

sessions I asked the students: "What do you want to do next week - another lab session or a classic pen and paper exercise session?". I was surprised by the answer: about 70% of the students wanted the classic pen and paper session. I asked them if they did not find the labs useful but then about 95% all said "no I found the lab sessions very useful". I suggested to the students that we'd have a lab session (i.e. everyone would be physically in the lab) but it would not be compulsory to do the lab sheet in other words that the students could choose to just do pen and paper exercises (the same tutors who would help with the computer package, could help with the pen and paper stuff). Everyone seems quite happy with that suggestion. On reflection I think it's a very good idea as firstly it will cater to both student types. Secondly the "point" of using the computer package is to help them learn so by perhaps placing them in front of the tool they might use it "because they need it" and not "because I've told them to use it". This is a basic "concrete" example of what I'll be doing in my lessons but I completely agree that in general I must not assume that everyone learns (or should learn) like I do. Paul (my mentor) read an earlier draft of this journal and commented that he felt very similar, he wondered if this was perhaps true of most mathematics lecturers. I suppose in a way it would be, we were mostly "good students" who "got it" so I think we probably often assume that others should "get it". Perhaps that's why mathematics is often taught in a dry manner...

On reflection all of the above seems to confirm that I am an individual learner, perhaps this is further confirmed by my relatively stochastic upbringing where it is conceivable to imagine that a big reliance on a teacher would have resulted in failure as teachers were constantly changing as I moved? In my next journal I will look into the ramification of being a student centred teacher as far as lesson planning is concerned following up some of the ideas presented in [7].

As an individual learner I must make sure that I am not an individual teacher. I look forward to further looking into learning styles and teaching techniques that will ensure that I don't leave every student at the doorway. Perhaps some will need me to walk through the door with them.

Word Count:1568

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So what for my lesson planning?

VK

The ILO for this journal:

- 1. Locate yourself in your discipline, your department, your University and wide national and international contexts.**
- 2. Explore the relationship between research, scholarship, related professional activities and teaching and learning as relevant to your own teaching practice.**

This is my final journal for Module 1. The aim of this journal is to synthesise my reflections from the previous journals to try and understand what I think it all means with regards to my lesson planning going forward through PUCTL and ultimately my teaching career.

The immediate benefit I gained from the three day workshop was the importance of lesson planning [2]. In the past the most planning I would undertake was to simply match the teaching notes to the syllabus. I plan on thinking about ILOs in all future classes I teach. This is not only due to the ILOs of this particular PCUTL module but also because it enables me to better structure the lessons I plan on teaching. I'll return to the use of ILOs when it comes to lesson planning towards the end of this journal.

In my first journal I spent a while reflecting on "what makes studying mathematics at Cardiff special". Being part of the Operational Research (OR) group made that a relatively simple task. At Cardiff the undergraduate programme contains more OR modules than other programmes in the UK and as such places it well on the agenda of employability.

In my second journal I concentrated on my scores from the various tests I undertook at the beginning of this module. I of course understand that these tests come with a major health warning and are simply meant to be indicative. As such I tried to look at them in a critical fashion trying to identify the sort of learning and teacher I was whilst being guided by the tests. The main thing I take from that journal is that I've realised that I am a very student structured learner. I was also able to localise myself on the Kolb cycle [5] and using [4] my corresponding mathematical learning style: an Analyzer and a Synthesizer.

I'll come back to what both of these conclusions mean with respect to my lesson planning and my students after thinking a bit more about "classic" methods of teaching in Mathematics.

In line with the Mathematical Subject Board indicators [6] most teaching of Mathematics is done through lectures and assessed through exams. I am obviously making a wide generalisation and stereotyping a lot of Mathematicians who are excellent teachers and for whom I have a lot of respect but my reasons for doing so will hopefully become clear. I think that when someone starts critiquing this as a methodology they are (potentially) in danger of trying to "fix something

that is not broken". This of course does not mean that one shouldn't be open to critique and importantly potential improvement, indeed the above methodology might actually be broken (who knows?). With regards to assessment of mathematics for example in [3] a discussion is given of various other techniques that can be used to assess mathematics (group projects, coursework) however I still believe that examination is often the most appropriate method (although there are of course natural exceptions).

With regards to teaching methodologies, lecturing is by far the classic methodology and it was the way I learnt. As I say in journal 2 this is also (by the time they get to University) what a lot of students are used to. Having said that upon reflection I think a big reason for this is that the people who lecture are the people for whom that system worked. As such there is a major risk that this methodology does not work for everybody. It is ultimately very important to ensure that all learning style are catered for. In particular the lecture based approach runs a risk of encouraging surface learning as mentioned in [1].

One of the major benefits (as far as personal development is concerned) of this module was the fact that I was reviewed by my mentor. I have always enjoyed being as open as possible with my performances so as to ensure they are as good as they can be (this goes by to the way young kids are taught to play rugby I believe, we're taught that feedback is never a negative thing no matter the form). I have started trying to keep this in mind with regards to my research for example where I've started putting my research code online (whilst I'm working on it and when it's possible) and discuss it openly on social networks. This is a growing trend in mathematics and one that I think is to be commended. I've already had one or two helpful comments with regards to some code I've been writing (the comments in fact pointed out embarrassing mistakes but this was of obvious benefit). So I was very much looking forward to having my mentor sit in on one of my lectures.

The lecture we picked was one that lends itself very well to role playing and student participation. This is something I am particularly fond of and was very much looking forward to teaching the students. Interestingly the first thing I wrote on the board was a mistake, I immediately realised that I was nervous having my mentor there (which I think is normal). In a way it was good to realise that I was nervous so that I could "shake it off" and just teach in a normal fashion. The lecture went well I believe (see the review and response for further details) and in particular I was able to use a majority of methods such as videos which were at the same point humorous and instructive. I also used Sage (an open source mathematical package) when needed during the lecture, this is most certainly linked to the fact that I am a visual learner. Sage enabled me to quickly demonstrate/visualise mathematical concepts to the students (I will come back to the point about Sage being open source at the very end of this journal). Finally I also played various role playing games. This was a very natural and worthy exercise in the particular subject chosen (it caters well to the concrete experience part of the Kold cycle). Importantly these aspects were naturally embedded throughout a classic Mathematics lecture (i.e. lecture with white board and notes which ensured other parts of the cycle were met). I was glad to see that my mentor and I were in agreement that this catered to all learning styles.

The general format of the lecture I think was a good one. The thing I will be concentrating on is how to mainstream this methodology. Indeed with certain subjects there is no immediate role playing games that comes to mind. Further to this videos are not always useful during the actual lecture (notions of flipped classroom [8] is something I have explored in other modules and something I am a huge fan of). If I find a way to consistently mainstream these activities I think that this will lessen the risk of surface learning from the students. Indeed simply the fact that they have to stand up to play some of the games is a huge positive as it at least ensures that they are relatively awake! Today (12/10/2012) I have spent some time planning a role playing game of sorts for a subject matter that is perhaps not as easy to associate. I look forward to trying that on the 16/10/2012 to see if it works. I also believe that by concentrating on the ILOs for each lecture it should allow me to ensure that these extra-whiteboard activities are used correctly. If anything realising that that is something I want to do has been a huge benefit of this module.

Another topic that I am keen to develop is the use of a mathematical computer packages in teaching and learning. This is not straightforward. In general courses might be completely computational (numerical analysis, simulation) in which case the use of a computer package is straightforwardly implemented "by design". The difficulty lies in using such packages to aid students to prepare for what will eventually be a "pen and paper" exam (which is an ideal assessment for certain ILOs). I recently had a conversation with some academics (who have much more experience than me in the use of the package considered) about how to best implement such packages in teaching [10]. My thoughts on the subject is that one must make the package useful to the student and I propose three phases of use:

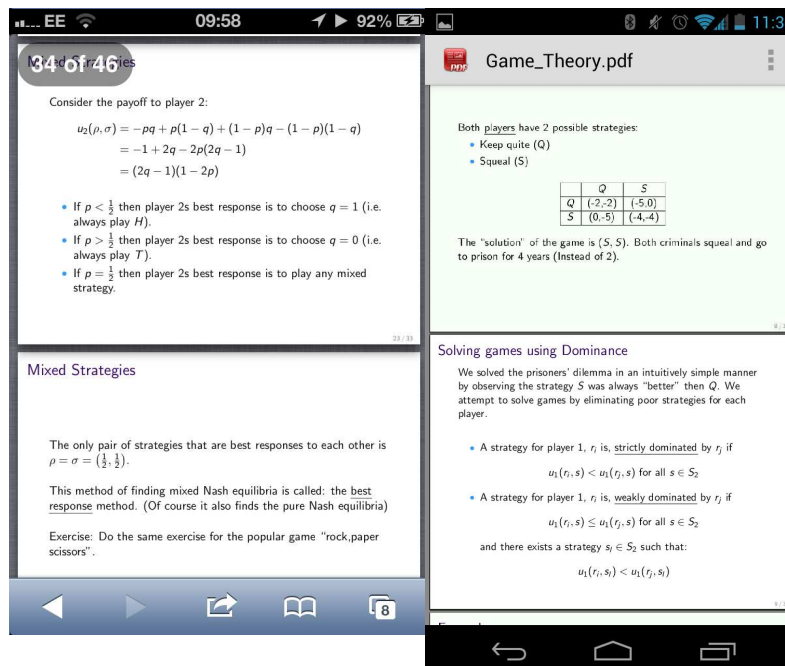
- Seeing
- Familiarising
- Using

During the first phase I make use of the package during lectures but not expecting the students to "know what I'm doing" i.e. I just want the students to see the benefit. The "Familiarising" phase corresponds to the students using the package but without needing much input and/or knowledge (the Sage interact website is ideal for this [7]). Finally the students are taught how to use the package. This format seems to have worked this year but I plan to spend more time thinking about this as I feel that the use of a computer package will be particularly suited to the Allegorizers and Synthesizers. It is also an important skill for students to gain as far as employability is concerned.

My general plan of action so to speak with my lesson planning is to ensure I cater for all learning types. As discussed in my previous journal I am a very student centred learner. To reference [9] I feel that this implies that I naturally have a Learning centred orientation as opposed to a Teaching centred orientation. Classic mathematical teaching (Lecture + Whiteboard) is a Teaching centred approach and as such I feel that it will also be natural for me to use classic methods to cater for students who need more of a Teaching centred approach. I feel that I managed this quite well in the class my mentor reviewed. The difficulty will be ensuring I

mainstream these methodologies to further subjects as well as keep an open mind to future technologies that students will expect to be able to use. This is something I'll be concentrating on in future modules.

One final aspect that I have not considered directly anywhere else in my journals is the aspect of inclusion and diversity. The main consideration from the point of view of a lecturer is to ensure that students are able to access and understand teaching materials (notes etc). In mathematics this needs to be considered from the point of view of students with disabilities that might make reading of notes difficult. A further issue to consider is software, in this day and age of digital technologies I think it is important to ensure that we do not expect students to have to have any software that is not freely available to them. As such the primary source of delivery of my research materials is in portable document format (pdfs). This enabled students to zoom in as required to ensure that they can make the text as large as they need to be. With regards to software, pdfs are a uniform format (as opposed to Microsoft Word for example which requires the purchase of Microsoft Office) that can be opened with a variety of freely available software. Furthermore with mathematical texts distributing the notes in formats other than pdf would require compatibility of software (for example the particular version of Microsoft Word being used and/or Mathtype or other). Finally in this modern day and age with individuality of consumption of digital content through smart phones and other sources I think that I must ensure that I am using formats that can be accessed through any medium. Here is a screenshot of some of my notes being viewed on smartphones (note that this is not restricted to any particular smartphones as opposed to certain apps for example):



The color of text and background in my notes are chosen to be as basic as possible (black on white) but it is my understanding that some disabilities might make various colour combinations easier to read. As such I plan to also make available the LaTeX source code for all my notes

so that student might modify them if they wished to (note again that this doesn't require the purchase of any software).

Another point of view to be considered is that of international students whose English might be weak (recalling the fact that growing an international student base is high on the agenda of Cardiff University). In a subject like mathematics this is not as big a problem (mathematics is ultimately a universal language) however difficulties might arise during my lectures where students do not perhaps understand me. I plan on further exploring ideas linked to flipped classroom methodologies (I already have a few video clips that explain certain concepts). Being able to watch a video of lecture topics with subtitles that can be repeated as needed is something I believe would be useful to all students (not just those with weak levels of English).

I have a lot of things to think about with regards to the rest of PCUTL. I think this module has helped me understand what I'd like my lessons to contain, the question arises of just exactly how to make sure that I do this throughout all my courses.

Word Count: 2222

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Some concluding thoughts

I hope that the main theme of PCUTL for me will be to ensure that I “create learning opportunities”. This term (Autumn 2012) I taught the first four weeks of a module that taught a variety of analytical methods to MSc students. Throughout this period I “gave a variety of opportunities” to students through the use of videos, computer programs and teaching notes that were available to students before and after lectures. My students were kind enough to fill in some feedback with regards to how they used these resources. I look forward to analysing this further in Module 2. Another teaching model I hope to explore is the use of “Inquiry Based Learning” or “Problem Based Learning”. One way I plan on doing this is through the use of simple problems distributed to students prior to a lesson. I’ll ask students to present their solutions to the problem to the rest of the class at the beginning of each lecture. There are various goals to this:

- Improve their presentation skills.
- Make active learning of the students.
- Make students more aware of the learning opportunities available to them through a hybrid of the classic “flipped classroom”/“Inquiry Based Learning” models.

Teaching resources

This contains a list of the various teaching resources relevant to my PRLT:

- Hard copies (included in this portfolio):
 - Lecture slides used.
 - Exercise sheet distributed.
 - Solutions to exercise sheet.
 - Sage lab sheet.
 - Solutions to Sage lab sheet.
- Sage code snippets (interactive programmes that can be run on any modern browser - some issues arise on Internet Explorer):
 - 2 by 2 normal form games: <http://interact.sagemath.org/node/49>.
- Videos shared with students prior to lecture:
 - A research talk which makes use of game theory: <http://youtu.be/w9CGYN8uErI>.
 - A brief video introducing mixed strategies: <http://youtu.be/poYucyX7-gE>.
- Videos used during the lecture:
 - A clip from the movie “A beautiful mind”: <http://youtu.be/or65M4Ht4Kk>.
 - Two clips from the TV game show “Golden Balls”: <http://youtu.be/p3Uos2fzIJO> and <http://youtu.be/S0qjK3TWZE8>.
- Finally a blog post that I wrote describing one of the games played in class (this was subsequently shared with the students): <http://goo.gl/oHoz0>.

Links to all of the above can be found at my personal website (specifically under the Teaching/MAT001/Game Theory sub site):

www.vincent-knight.com

Game Theory

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Overview

Normal Form Games

Pure Nash Equilibrium

Mixed Nash Equilibrium

Normal Form Games

Game Theory: Introduction

Often decision analysis does not only depend on chance but on the decisions made by others: interactive decision problems.

Such decision problems are called games. The individuals making the decisions are called players.

2 Player Static Games

2 Player Static Games

We shall consider 2 player static games. Assume two players have two sets of available strategies: $S_1 = \{r_1, \dots, r_m\}$ and $S_2 = \{s_1, \dots, s_n\}$. Let $u_1(r, s)$, $u_2(r, s)$ be the utility gained by player 1 and 2 for a pair of strategies (s, r) .

	s_1	s_2	\dots	s_n
r_1	(u_1, u_2)	(u_1, u_2)	\dots	(u_1, u_2)
r_2	(u_1, u_2)	(u_1, u_2)	\dots	(u_1, u_2)
\vdots	\vdots	\vdots	\ddots	\vdots
r_m	(u_1, u_2)	(u_1, u_2)	\dots	(u_1, u_2)

Both players aim to choose from their available strategies so as to maximise u_1 and u_2 .

Example: Prisoner's Dilemma

Two criminal suspects have been caught. They have been isolated and are being questioned separately by the police. The following offer is made to both suspects:

- If one confesses that they both committed the crime then the confessor will be set free and the other will spend 5 years in jail.
- If both confess, then they will each get a 4 year sentence.
- If neither confess, then they will each spend 2 years in jail.

Example: Prisoner's Dilemma

Both players have 2 possible strategies:

- Keep quite (Q)
- Squeal (S)

	Q	S
Q	(-2,-2)	(-5,0)
S	(0,-5)	(-4,-4)

The “solution” of the game is (S, S). Both criminals squeal and go to prison for 4 years (Instead of 2).

Solving games using Dominance

We solved the prisoners' dilemma in an intuitively simple manner by observing the strategy S was always “better” than Q . We attempt to solve games by eliminating poor strategies for each player.

- A strategy for player 1, r_i is, strictly dominated by r_j if

$$u_1(r_i, s) < u_1(r_j, s) \text{ for all } s \in S_2$$

- A strategy for player 1, r_i is, weakly dominated by r_j if

$$u_1(r_i, s) \leq u_1(r_j, s) \text{ for all } s \in S_2$$

and there exists a strategy $s_l \in S_2$ such that:

$$u_1(r_i, s_l) < u_1(r_j, s_l)$$

Example

Consider the following game:

	s_1	s_2
r_1	(3, 3)	(2, 2)
r_2	(2, 1)	(2, 1)

For player 2, s_1 weakly dominates s_2 . For player 1, r_1 weakly dominates r_2 . Thus (r_1, s_1) is the “solution” of this game.

Common Knowledge of Rationality

To solve a game by elimination of dominated strategies we have to assume that the players are rational. However, we can go further, if we also assume that:

- The players are rational.
- The players all know that the other players are rational.
- The players all know that the other players know that they are rational.
- ...

This chain of assumptions is called Common Knowledge of Rationality (CKR). By applying the CKR assumption, we can try to solve games by iterating the elimination of dominated strategies.

Example

	s_1	s_2	s_3
r_1	(1, 0)	(1, 2)	(0, 1)
r_2	(0, 3)	(0, 1)	(2, 0)

Initially player 1 has no dominated strategies. For player 2, s_3 is dominated by s_2 . **Now**, r_2 is dominated by r_1 . **Finally**, s_1 is dominated by s_2 . Thus (r_1, s_2) is the “solution” of this game.

Pure Nash Equilibrium

(Pure) Nash Equilibrium

Importantly, certain games cannot be solved using the iterated elimination of dominated strategies:

	s_1	s_2	s_3
r_1	(10, 0)	(5, 1)	(4, -2)
r_2	(10, 1)	(5, 0)	(1, -1)

	s_1	s_2	s_3
r_1	(1, 3)	(4, 2)	(2, 2)
r_2	(4, 0)	(0, 3)	(4, 1)
r_3	(2, 5)	(3, 4)	(5, 6)

(exercise: why does iterated elimination fail here?)

Nash Equilibrium

A (pure) Nash equilibrium is a pair of strategies (\tilde{r}, \tilde{s}) such that

$$u_1(\tilde{r}, \tilde{s}) \geq u_1(r, \tilde{s}) \text{ for all } r \in S_1$$

and

$$u_2(\tilde{r}, \tilde{s}) \geq u_2(\tilde{r}, s) \text{ for all } s \in S_2$$

Testing for Nash Equilibrium

One can find Nash equilibria by checking all strategy pairs and seeing if either player can improve their outcome.

	s_1	s_2	s_3
r_1	(10, 0)	(5, 1)	(4, -2)
r_2	(10, 1)	(5, 0)	(1, -1)

Nash Equilibria need not be unique!

Best response strategies

A strategy for player 1 r^* is a best response to some fixed strategy for player 2, s if:

$$u_1(r^*, s) \geq u_1(r, s) \text{ for all } r \in S_1$$

A strategy for player 2 s^* is a best response to some fixed strategy for player 1, r if:

$$u_2(r, s^*) \geq u_2(r, s) \text{ for all } s \in S_2$$

To use this definition to find Nash Equilibria we find for each player, the set of best responses to every possible strategy of the other player. We then look for pairs of strategies that are best responses to each other.

Example

	s_1	s_2	s_3
r_1	(1, 3)	(4, 2)	(2, 2)
r_2	(4, 0)	(0, 3)	(4, 1)
r_3	(2, 5)	(3, 4)	(5, 6)

Mixed Nash Equilibrium

Mixed Strategies

Importantly some games do not have **pure** Nash equilibria!
Consider the following game:

Two players each place a coin on a table, either “heads up” (strategy H) or “tails up” (strategy T). If the pennies match, player 1 wins, if the pennies differ, then player 2 wins.

	H	T
H	$(1, -1)$	$(-1, 1)$
T	$(-1, 1)$	$(1, -1)$

Mixed Strategies

In order to solve such games, we need to consider mixed strategies. I.e. we attach a distribution to the set of strategies of each player.

In the matching pennies example, let $\rho = (p, 1 - p)$ be the mixed strategy for player 1. I.e. player 1 plays H with probability p and plays T with probability $1 - p$.

Similarly let $\sigma = (q, 1 - q)$ be the mixed strategy for player 2. I.e. player 2 plays H with probability q and plays T with probability $1 - q$.

Mixed Strategies

Consider the payoff to player 1:

$$\begin{aligned}u_1(\rho, \sigma) &= pq - p(1 - q) - (1 - p)q + (1 - p)(1 - q) \\ &= 1 - 2q + 2p(2q - 1) \\ &= (2q - 1)(2p - 1)\end{aligned}$$

- If $q < \frac{1}{2}$ then player 1's best response is to choose $p = 0$ (i.e. always play T).
- If $q > \frac{1}{2}$ then player 1's best response is to choose $p = 1$ (i.e. always play H).
- If $q = \frac{1}{2}$ then player 1's best response is to play any mixed strategy.

Mixed Strategies

Consider the payoff to player 2:

$$\begin{aligned}u_2(\rho, \sigma) &= -pq + p(1 - q) + (1 - p)q - (1 - p)(1 - q) \\ &= -1 + 2q - 2p(2q - 1) \\ &= (2q - 1)(1 - 2p)\end{aligned}$$

- If $p < \frac{1}{2}$ then player 2's best response is to choose $q = 1$ (i.e. always play H).
- If $p > \frac{1}{2}$ then player 2's best response is to choose $q = 0$ (i.e. always play T).
- If $p = \frac{1}{2}$ then player 2's best response is to play any mixed strategy.

Mixed Strategies

The only pair of strategies that are best responses to each other is $\rho = \sigma = (\frac{1}{2}, \frac{1}{2})$.

This method of finding mixed Nash equilibria is called: the best response method. (Of course it also finds the pure Nash equilibria)

Exercise: Do the same exercise for the popular game “rock,paper scissors” .

Example

	s_1	s_2
r_1	$(0, 0)$	$(2, 1)$
r_2	$(1, 2)$	$(0, 0)$

As before:

$$u_1(\rho, \sigma) = q + p(2 - 3q)$$

$$u_2(\rho, \sigma) = p + q(2 - 3p)$$

Best responses for player 1:

$$\rho^* = \begin{cases} (0, 1) & \text{if } q > \frac{2}{3} \\ (1, 0) & \text{if } q < \frac{2}{3} \\ (x, 1 - x) \text{ with } 0 \leq x \leq 1 & \text{if } q = \frac{2}{3} \end{cases}$$

Example

	s_1	s_2
r_1	(0, 0)	(2, 1)
r_2	(1, 2)	(0, 0)

As before:

$$u_1(\rho, \sigma) = q + p(2 - 3q)$$

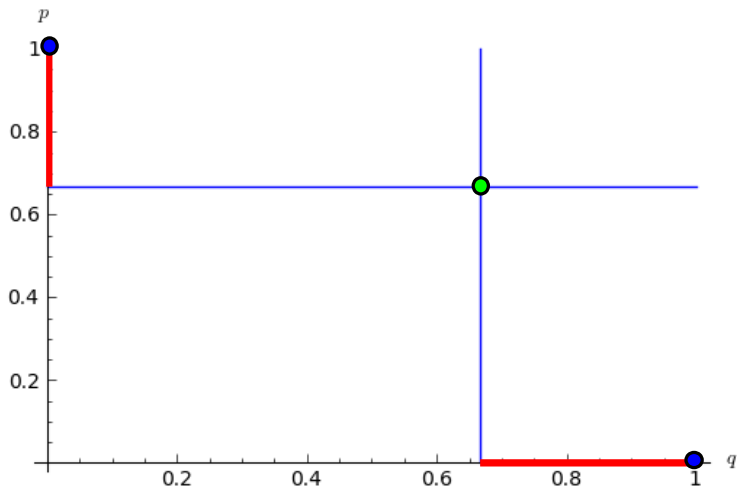
$$u_2(\rho, \sigma) = p + q(2 - 3p)$$

Best responses for player 2:

$$\sigma^* = \begin{cases} (0, 1) & \text{if } p > \frac{2}{3} \\ (1, 0) & \text{if } p < \frac{2}{3} \\ (y, 1 - y) \text{ with } 0 \leq y \leq 1 & \text{if } p = \frac{2}{3} \end{cases}$$

Example

We plot both best responses:



Example

Thus for this example there are 3 Nash equilibria:

$$(r_1, s_2), (r_2, s_1) \text{ and } (\rho, \sigma) \text{ with } \rho = \sigma = \left(\frac{2}{3}, \frac{1}{3}\right)$$

Equality of Payoffs

The support of a strategy ρ is the set $S(\rho)$ of all strategies for which ρ has non zero probability.

For example, if the strategy set is $\{A, B, C\}$ then the support of the mixed strategy $(\frac{1}{3}, \frac{2}{3}, 0)$ is $\{A, B\}$. Similarly the support of the mixed strategy $(\frac{1}{2}, 0, \frac{1}{2})$ is $\{A, C\}$.

This leads to a very powerful result.

Equality of Payoffs Theorem

Let (ρ, σ) be a Nash equilibrium, and let S_1^* be the support of ρ .
Then:

$$u_1(\rho, \sigma) = u_1(r, \sigma) \text{ for all } r \in S_1^*$$

Equality of Payoffs

Consider the matching pennies game. Let σ be the mixed strategy of player 2 with a chance of playing H of q and a chance of playing T with probability $(1 - q)$. From the Equality of Payoffs theorem we have:

$$u_1(H, \sigma) = u_1(T, \sigma)$$

$$qu_1(H, H) + (1 - q)u_1(H, T) = qu_1(T, H) + (1 - q)u_1(T, T)$$

$$q - (1 - q) = -q + (1 - q)$$

$$q = \frac{1}{2}$$

Equality of Payoffs

Let ρ be the mixed strategy of player 1 with a chance of playing H of p and a chance of playing T with probability $1 - p$. From the Equality of Payoffs theorem we also have:

$$u_2(\rho, H) = u_2(\rho, T)$$

$$pu_2(H, H) + (1 - p)u_2(T, H) = pu_2(H, T) + (1 - p)u_2(T, T)$$

$$-p + (1 - p) = p - (1 - p)$$

$$p = \frac{1}{2}$$

As expected.

Nash's Theorem

Every game that has a finite set of strategies has at least one Nash equilibrium (involving pure or mixed strategies).

(It can be shown that there is always an odd number of Nash equilibria.)

Game Theory Exercise Sheet

This sheet was updated on October 1, 2012.

- Find the pure Nash equilibria for the following games:

	s_1	s_2	s_3
r_1	(6, 3)	(2, 2)	(2, 2)
r_2	(4, 0)	(0, 3)	(4, 5)
r_3	(2, 3)	(3, 4)	(3, 2)

(a)

	s_1	s_2
r_1	(7, -2)	(4, 0)
r_2	(1, -5)	(0, -4)
r_3	(4, -1)	(3, -5)
r_4	(6, -7)	(4, -5)

(b)

	s_1	s_2	s_3
r_1	(160, 2)	(205, 2)	(44, 2)
r_2	(175, 1)	(180, .5)	(45, 5)
r_3	(201, 3)	(204, 4)	(50, 10)
r_4	(120, 4)	(107, 6)	(49, 2)

(c)

	s_1	s_2	s_3
r_1	(0, 0)	(-1, 1)	(1, -1)
r_2	(1, -1)	(0, 0)	(-1, 1)
r_3	(-1, 1)	(1, -1)	(0, 0)

(d)

For which games do you suspect not having identified all equilibria and why?

- The following game is known as the *traveller's dilemma*:

An airline loses two suitcases belonging to two different travelers. Both suitcases happen to be identical and contain identical antiques. An airline manager tasked to settle the claims of both travelers explains that the airline is liable for a maximum of 100 per suitcase, and in order to determine an honest appraised value of the antiques the manager separates both travelers so they can't confer, and asks them to write down the amount of their value at no less than 2 and no larger than 100. He also tells them that if both write down the same number, he will treat that number as the true dollar value of both suitcases and reimburse both travelers that amount. However, if one writes down a smaller number than the other, this smaller number will be taken as the true dollar value, and both travelers will receive that amount along with a bonus/malus: 2 extra will be paid to the traveler who wrote down the lower value and a 2 deduction will be taken from the person who wrote down the higher amount. The challenge is: what strategy should both travelers follow to decide the value they should write down?

- Identify all the Nash equilibria for the classic game: *Rock, Paper, Scissors*.
- Assume a fighter must find a bomb being transported on two different bombers. The two bombers fly in such a way such that the guns of bomber 2 gives more protection to bomber 1 than the guns of bomber 1 give to bomber 2. I.e. bomber 1 is the best protected plane:
 - Bomber 1 has a 80% chance of surviving an attack.
 - Bomber 2 has a 60% chance of surviving an attack

The bomber must decide which plane to use to transport the bomb. The fighter must choose which plane to attack. The bi-matrix representation of this game is given below:

	Attack Bomber 1	Attack Bomber 2
Transport with Bomber 1	(80, -80)	(100, -100)
Transport with Bomber 2	(100, -100)	(60, -60)

Identify all the Nash equilibria for this game.

5. Using the equality of payoffs theorem identify all the Nash equilibria for the following games:

	s_1	s_2
r_1	(0, 0)	(2, 1)
r_2	(1, 2)	(0, 0)

(a)

	s_1	s_2
r_1	(3, 3)	(3, 2)
r_2	(2, 2)	(5, 6)
r_3	(0, 3)	(6, 1)

(b)

6. Assume two pedestrians are walking on the same sidewalk, towards each other. Both pedestrians have two options to ensure they do not walk in to each other:

- Step left
- Step right

- (a) Give a bi-matrix representation of this system.
- (b) By clearly stating the technique you use, identify all pure Nash equilibria for this game.
- (c) Using the equality of payoffs theorem, identify all Nash equilibria for this game.

Game Theory Exercise Sheet SOLUTIONS

This sheet was last updated on October 15, 2012.

1.

	s_1	s_2	s_3
r_1	(<u>6</u> , <u>3</u>)	(2, 2)	(2, 2)
r_2	(4, 0)	(0, 3)	(<u>4</u> , <u>5</u>)
r_3	(2, 3)	(<u>3</u> , <u>4</u>)	(3, 2)

(a)

	s_1	s_2
r_1	(<u>7</u> , -2)	(<u>4</u> , <u>0</u>)
r_2	(1, -5)	(0, <u>-4</u>)
r_3	(4, <u>-1</u>)	(3, -5)
r_4	(6, -7)	(<u>4</u> , <u>-5</u>)

(b)

	s_1	s_2	s_3
r_1	(160, 2)	(<u>205</u> , <u>2</u>)	(44, <u>2</u>)
r_2	(175, 1)	(180, .5)	(45, <u>5</u>)
r_3	(<u>201</u> , 3)	(204, 4)	(<u>50</u> , <u>10</u>)
r_4	(120, 4)	(107, <u>6</u>)	(49, 2)

(c)

	s_1	s_2	s_3
r_1	(0, 0)	(-1, 1)	(1, -1)
r_2	(1, -1)	(0, 0)	(-1, 1)
r_3	(-1, 1)	(1, -1)	(0, 0)

(d)

Since the number of Nash Equilibria for any given game is odd, we expect to not have identified all equilibria for (b), (c) and (d).

2. The bi-matrix representation is given by:

	100	99	98	...	3	2
100	(100, 100)	(97, 101)	(96, 100)	...	(1, 5)	(0, 4)
99	(101, 97)	(99, 99)	(96, 100)	...	(1, 5)	(0, 4)
98	(100, 96)	(100, 96)	(98, 98)	...	(1, 5)	(0, 4)
\vdots	\ddots	\vdots	\vdots
3	(5, 1)	(5, 1)	(5, 1)	...	(3, 3)	(0, 4)
2	(4, 0)	(4, 0)	(4, 0)	...	(4, 0)	(2, 2)

This game is immediate to solve with dominance and so the Nash equilibrium is (2, 2).

3. We have the bi-matrix game representation:

	R	P	S
R	(0, 0)	(-1, 1)	(1, -1)
P	(1, -1)	(0, 0)	(-1, 1)
S	(-1, 1)	(1, -1)	(0, 0)

There is no pure Nash equilibrium and it is immediate to see that no mixed strategy will have support of size 2. Indeed, assume that a mixed strategy for player 1 does not play “scissors”. Player 2 would have an immediate benefit of playing the pure strategy “paper” (as he’ll never lose). This can be shown mathematically.

Thus the mixed strategy for player 1, ρ , will be of the form:

$$\rho = (p, q, 1 - p - q)$$

The mixed strategy for player 2, σ , will be of the form:

$$\sigma = (u, v, 1 - u - v)$$

Using the equality of payoffs theorem, we have:

$$u_1(R, \sigma) = u_1(S, \sigma) = u_1(T, \sigma) \tag{1}$$

and

$$u_2(\rho, R) = u_2(\rho, S) = u_2(\rho, T) \tag{2}$$

We have:

$$\begin{aligned} u_1(R, \sigma) &= -v + 1 - u - v \quad (a) \\ u_1(P, \sigma) &= u - 1 + u + v \quad (b) \\ u_1(S, \sigma) &= -u + v \quad (c) \end{aligned} \tag{3}$$

Combining (1) and (3) gives:

$$\begin{aligned} (a) = (b) &\Rightarrow 3u + 3v = 2 \\ (a) = (c) &\Rightarrow 3v = 1 \\ (b) = (c) &\Rightarrow 3u = 1 \end{aligned}$$

Thus $\sigma = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ as expected. A similar approach using (3) gives the expected result for ρ .

4. Recall:

	Attack Bomber 1	Attack Bomber 2
Transport with Bomber 1	(80, -80)	(100, -100)
Transport with Bomber 2	(100, -100)	(60, -60)

There is clearly no pure Nash equilibria. Let the bombers use bomber 1 with probability p (thus they use bomber 2 with probability $1 - p$). We denote the mixed strategy of the bombers by $\rho = \{p, 1 - p\}$. Let the fighter attack bomber 1 with probability q (thus the fighter attacks bomber 2 with probability $1 - q$). We denote the mixed strategy of the fighter by $\sigma = \{q, 1 - q\}$. We could use the equality of payoffs theorem to solve this problem. Let us however, consider a direct approach by looking at best responses:

$$\begin{aligned} u_1(\rho, \sigma) &= 80pq + 100(p(1 - q) + q(1 - p)) + 60(1 - q)(1 - p) \\ &= 20(3 + 2p + 2q - 3pq) \\ &= 20(p(2 - 3q) + 3 + 2q) \end{aligned}$$

We immediately see that:

- If $q < \frac{2}{3}$ then player 1s best response is to choose $p = 1$.
- If $q > \frac{2}{3}$ then player 1s best response is to choose $p = 0$.
- If $q = \frac{2}{3}$ then player 1s best response is to play any mixed strategy.

Similarly we have:

$$\begin{aligned} u_2(\rho, \sigma) &= -(80pq + 100(p(1 - q) + q(1 - p)) + 60(1 - q)(1 - p)) \\ &= -(20(3 + 2p + 2q - 3pq)) \\ &= 20(q(3p - 2) - 3 - 2p) \end{aligned}$$

and we have:

- If $p < \frac{2}{3}$ then player 1s best response is to choose $q = 0$.
- If $p > \frac{2}{3}$ then player 1s best response is to choose $q = 1$.
- If $p = \frac{2}{3}$ then player 1s best response is to play any mixed strategy.

The only strategies that are best responses to each other is $\rho = \sigma = (\frac{2}{3}, \frac{1}{3})$.

5. Using the equality of payoffs theorem identify all the Nash equilibria for the following games: (a)

	s_1	s_2
r_1	(0, 0)	(2, 1)
r_2	(1, 2)	(0, 0)

The pure Nash equilibria are given by (r_2, s_1) and (r_1, s_2) . Consider the mixed strategies $\rho = (p, 1 - p)$ and $\sigma = (q, 1 - q)$. By the equality of payoff theorem we have:

$$u_1(r_1, \sigma) = u_1(r_2, \sigma)$$

and

$$u_2(\rho, s_1) = u_2(\rho, s_2)$$

The first equation is equivalent to:

$$2(1 - q) = q$$

which gives $q = \frac{2}{3}$. Similarly we get $p = \frac{2}{3}$. Thus $\rho = \sigma = (\frac{2}{3}, \frac{1}{3})$.

(b)

	s_1	s_2
r_1	(3, 3)	(3, 2)
r_2	(2, 2)	(5, 6)
r_3	(0, 3)	(6, 1)

The pure Nash equilibria is (r_1, s_1) . Consider the mixed strategies $\rho = (p, q, 1 - p - q)$ and $\sigma = (u, 1 - u)$. The difficult part of this problem is to identify the various different supports that ρ may have (it is obvious that the size of the support of σ is 2). Let us first consider supports of size 2:

- Assume that the support of ρ is $\{r_1, r_2\}$:
Using the equality of payoffs theorem we have:

$$u_1(r_1, \sigma) = u_1(r_2, \sigma)$$

and

$$u_2(\rho, s_1) = u_2(\rho, s_2)$$

this gives:

$$u_1(r_1, \sigma) = u_1(r_2, \sigma) \Rightarrow 3(u + 1 - u) = 2u + 5(1 - u) \Rightarrow u = \frac{2}{3}$$

and (recalling that in this case we have $\rho = (p, 1 - p, 0)$)

$$u_2(\rho, s_1) = u_2(\rho, s_2) \Rightarrow 3p + 2(1 - p) = 2p + 6(1 - p) \Rightarrow p = \frac{4}{5}$$

Thus this support gives the mixed Nash equilibrium: $(\{\frac{4}{5}, \frac{1}{5}, 0\}, \{\frac{2}{3}, \frac{1}{3}\})$

- Assume that the support of ρ is $\{r_2, r_3\}$:
Using the equality of payoffs theorem we have:

$$u_1(r_2, \sigma) = u_1(r_3, \sigma)$$

and

$$u_2(\rho, s_1) = u_2(\rho, s_2)$$

this gives:

$$u_1(r_2, \sigma) = u_1(r_3, \sigma) \Rightarrow 2u + 5(1 - u) = 0u + 6(1 - u) \Rightarrow u = \frac{1}{3}$$

and (recalling that in this case we have $\rho = (0, q, 1 - q)$)

$$u_2(\rho, s_1) = u_2(\rho, s_2) \Rightarrow 3q + 3(1 - q) = 6q + (1 - q) \Rightarrow q = \frac{1}{3}$$

Thus this support gives the mixed Nash equilibrium: $(\{0, \frac{1}{3}, \frac{2}{3}\}, \{\frac{1}{3}, \frac{2}{3}\})$

- Assume that the support of ρ is $\{r_1, r_3\}$:
Using the equality of payoffs theorem we have:

$$u_1(r_1, \sigma) = u_1(r_3, \sigma)$$

and

$$u_2(\rho, s_1) = u_2(\rho, s_2)$$

this gives:

$$u_1(r_1, \sigma) = u_1(r_3, \sigma) \Rightarrow 3u + 3(1 - u) = 0u + 6(1 - u) \Rightarrow u = \frac{1}{2}$$

and (recalling that in this case we have $\rho = (p, 0, 1 - p)$)

$$u_2(\rho, s_1) = u_2(\rho, s_2) \Rightarrow 3p + 3(1 - p) = 2p + (1 - p) \Rightarrow p = 2$$

However, this last value is not consistent with probabilities! Thus, this support does not have a Nash equilibrium.

We are left with having to consider one last support: $\{r_1, r_2, r_3\}$. It should be apparent that this case will simplify to one of the previous cases. Thus, we have found all the Nash equilibria:

$$(r_1, s_1), \left(\left\{ \frac{4}{5}, \frac{1}{5}, 0 \right\}, \left\{ \frac{2}{3}, \frac{1}{3} \right\} \right) \text{ and } \left(\left\{ 0, \frac{1}{3}, \frac{2}{3} \right\}, \left\{ \frac{1}{3}, \frac{2}{3} \right\} \right)$$

6. (a) Assuming “walking in to each other” gives both players a utility of -1 and “avoiding each other” a utility of 1 , the bi matrix representation of this game is:

	L	R
L	$(1, 1)$	$(-1, -1)$
R	$(-1, -1)$	$(1, 1)$

where $L, ; R$ represent the step left and right strategies respectively.

- (b) Using best responses we have:

	L	R
L	$(\underline{1}, \underline{1})$	$(-1, -1)$
R	$(-1, -1)$	$(\underline{1}, \underline{1})$

thus the two pure Nash equilibria are $\{L, L\}$ and $\{R, R\}$.

- (c) Assume player 1, plays the mixed strategy $\rho = (p, 1 - p)$ and player 2 plays the mixed strategy $\sigma = (q, 1 - q)$. By the equality of payoffs theorem we have:

$$\begin{aligned} u_1(L, \sigma) &= u_1(R, \sigma) & \text{and} & & u_2(\rho, L) &= u_2(\rho, R) \\ q + (1 - q)(-1) &= q(-1) + (1 - q) & \text{and} & & p + (1 - p)(-1) &= p(-1) + 1 - p \\ q &= \frac{1}{2} & \text{and} & & p &= \frac{1}{2} \end{aligned}$$

thus $p = q = \frac{1}{2}$ The mixed Nash equilibria is $\left\{ \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right\}$