4. (a) Provide definitions for the following terms:

- Normal form game.
- Strictly dominated strategy.
- Weakly dominated strategy.
- Best response strategy.
- Nash equilibrium.

[5]

(b) Consider the following game:

$$\begin{pmatrix} (7,3) & (0,2) \\ (2,0) & (6,2) \end{pmatrix}$$

(i) By clearly stating the techniques used, obtain all (if any) pure Nash equilibria. [4]

(ii) Sketch the utilities to player 1 (the row player) assuming that the 2nd player (the column player) plays a mixed strategy: $\sigma_2 = (y, 1-y)$.

[2]

(iii) Sketch the utilities to player 2 (the column player) assuming that the 1st player (the row player) plays a mixed strategy: $\sigma_1 = (x, 1-x)$.

[2]

(iv) State, prove and use the Equality of Payoffs theorem to obtain all Nash equilibria for the game.

[6]

(v) Consider the same game with an extra strategy for the row player:

$$\begin{pmatrix} (7,3) & (0,2) \\ (3,1) & (3,1) \\ (2,0) & (6,2) \end{pmatrix}$$

By directly calculating the set of best response strategies B_1 for the row player, obtain all Nash equilibria for this new game. State any theorem(s) used.

[6]

5. This question considers evolutionary population games. Throughout, the following game is considered:

Road users in a given country can choose to drive on either the left (L) side or the right (R) side of the road. The strategy set in this game is $S = \{L, R\}$.

If all users drive on the same side of the road then no accidents will occur. If users drive on the opposite side and meet each other then they may have an accident.

Considering a population vertor $\chi = (x, 1 - x)$ where x is the proportion of the population using strategy L, the utilities are given by:

$$u(L, \chi) = 1 + x$$

and

$$u(R, \chi) = 1 + (1 - x)$$

(a) Define a stable strategy in a population game.

[2]

(b) State and prove a theorem giving a necessary condition for stable strategies. Use this theorem to obtain all potential evolutionary stable strategies in the described game.

[6]

(c) Define a post entry population.

[2]

(d) Define an evolutionary stable strategy.

[2]

(e) Obtain all evolutionary stable strategies for the described game.

[12]

(f) Offer an interpretation for the answer to question (e).

[1]

6. (a) Define a characteristic function game $G = (N, v)$.

[2]

(b) Define the Shapley value.

[2]

(c) Obtain the Shapley value for the following characteristic function games:

$$v_1(c) = \begin{cases} 6, & \text{if } c = \{1\} \\ 6, & \text{if } c = \{2\} \\ 7, & \text{if } c = \{3\} \\ 7, & \text{if } c = \{1,2\} \\ 7, & \text{if } c = \{2,3\} \\ 20, & \text{if } c = \{1,3\} \\ 40, & \text{if } c = \{1,2,3\} \end{cases}$$

$$v_2(c) = \begin{cases} 100, & \text{if } c = \{1\} \\ 6, & \text{if } c = \{2\} \\ 7, & \text{if } c = \{3\} \\ 100, & \text{if } c = \{1,2\} \\ 7, & \text{if } c = \{2,3\} \\ 100, & \text{if } c = \{1,3\} \\ 100, & \text{if } c = \{1,2,3\} \end{cases}$$

[8]

(d) For a given characteristic function game $G = (N, v)$ a payoff vector λ is efficient if:

$$\sum_{i=1}^{N} \lambda_i = v(\Omega)$$

Prove that the Shapley value is efficient.

[6]

(e) For $G(N, v)$ a payoff vector λ is symmetric if the following holds:
If $v(C \cup i) = v(C \cup j)$ for all $C \in 2^{\Omega} \setminus \{i, j\}$ then $x_i = x_j$.
Prove that the Shapley value is symmetric.

[7]