

Markov Chains Sage Sheet

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1. Discrete Markov Chains

Navigate to the Sage code snippet at: <https://gist.github.com/drvinceknight/7b479fd1cd0868e4590c> and start a new Sage worksheet so that you can try a few of your own Sage commands.

- (a) Using the default inputs, what is the steady state distribution associated with this Markov chain (try and use the Sage “solve” command to verify this)?
- (b) How long does it seem to take to arrive at that state (try and use Sage to verify this)?

2. Continuous Markov Chains

The evolution of a Continuous Markov Chain obeys the following differential equation:

$$\frac{d\pi(t)}{dt} = \pi(t)Q$$

which has solution:

$$\pi(t) = \pi(0)e^{Qt}$$

This requires the calculation of the exponential of a Matrix. This is not straightforward! One approach is to use the following formula:

$$e^M = \mathbb{I} + \sum_{k=1}^{\infty} \frac{M^k}{k!}$$

Navigate to the Sage code snippet at <https://gist.github.com/drvinceknight/7d0094eeae4aa4508c6> and experiment with the code to see how good this approach is.

- (a) Consider the following Markov Chain:

$$Q = \begin{pmatrix} -3 & 3 & 0 & 0 \\ 4 & -7 & 3 & 0 \\ 0 & 4 & -7 & 3 \\ 0 & 0 & 4 & -4 \end{pmatrix}$$

- (b) Use the Sage “solve” command to obtain the steady state probabilities for this Markov Chain.