Markov Chains Exercise Sheet

Last updated: October 15, 2012.

1. Assume that a student can be in 1 of 4 states:

   - Rich
   - Average
   - Poor
   - In Debt

   Assume the following transition probabilities:

   - If a student is Rich, in the next time step the student will be:
     - Average: .75
     - Poor: .2
     - In Debt: .05
   - If a student is Average, in the next time step the student will be:
     - Rich: .05
     - Average: .2
     - In Debt: .45
   - If a student is Poor, in the next time step the student will be:
     - Average: .4
     - Poor: .3
     - In Debt: .2
   - If a student is In Debt, in the next time step the student will be:
     - Average: .15
     - Poor: .3
     - In Debt: .55

   Model the above as a discrete Markov chain and:

   (a) Draw the corresponding Markov chain and obtain the corresponding stochastic matrix.

   (b) Let us assume that a student starts their studies as "Average". What will be the probability of them being "Rich" after 1,2,3 time steps?

   (c) What is the steady state probability vector associated with this Markov chain?

2. Consider the following matrices. For the matrices that are stochastic matrices, draw the associated Markov Chain and obtain the steady state probabilities (if they exist, if not, explain why).

$$\begin{pmatrix}0 & 1\\ 0 & 1\end{pmatrix}$$
(a)

$$\begin{pmatrix}.5 & .25 & .25\\ 1 & 0 & 0\\ 0 & .23 & .77\\ .8 & .1 & .1\end{pmatrix}$$
(b)

$$\begin{pmatrix}\frac{1}{n} & \frac{n-1}{n}\\ \frac{n-1}{n} & \frac{1}{n}\end{pmatrix}$$
(c)

$$\begin{pmatrix}.2 & .3 & .5\\ .1 & .1 & .8\\ .7 & .1 & .2\end{pmatrix}$$
(d)

$$\begin{pmatrix}.2 & .3 & .1 & .4\\ 0 & .3 & .7 & 0\\ .5 & .2 & .1 & .2\\ .1 & 0 & 0 & .9\end{pmatrix}$$
(e)

$$\begin{pmatrix}.2 & .3 & .5\\ .3 & -.3 & 1\\ .2 & .2 & .6\end{pmatrix}$$
(f)

$$\begin{pmatrix}.5 & .5 & 0 & 0\\ 0 & .5 & .5 & 0\\ 0 & 0 & .5 & .5\\ .5 & 0 & 0 & .5\end{pmatrix}$$
(g)

$$\begin{pmatrix}\alpha & \beta\\ \omega & \gamma\end{pmatrix}$$
(h)

3. Consider the following (incomplete) transition matrix:

$$\begin{pmatrix}? & 2 & 1.5 & .5\\ ? & -5 & 1 & 3\\ 5 & 2 & ? & 1\\ 1 & ? & 1 & -2\end{pmatrix}$$

   (a) Fill in the missing values in the transition matrix.
   (b) Draw the Markov chain.
   (c) Obtain the steady state probabilities.

4. Consider the following matrices. For the matrices that are transition rate matrices, draw the associated Markov Chain and obtain the steady state probabilities (if they exist, if not, explain why).

$$\begin{pmatrix}-1 & 1\\ -1 & 1\end{pmatrix}$$
(a)

$$\begin{pmatrix}-5 & 0 & 5\\ 2 & 0 & -2\\ 0 & 3 & -3\\ 0 & 0 & 0\end{pmatrix}$$
(b)

$$\begin{pmatrix}-3 & 3 & 0\\ 0 & -3 & 3\\ 3 & 0 & -3\end{pmatrix}$$
(c)

$$\begin{pmatrix}-a & a & 0\\ b & -(a+b) & a\\ 0 & b & -b\end{pmatrix}$$
(d)

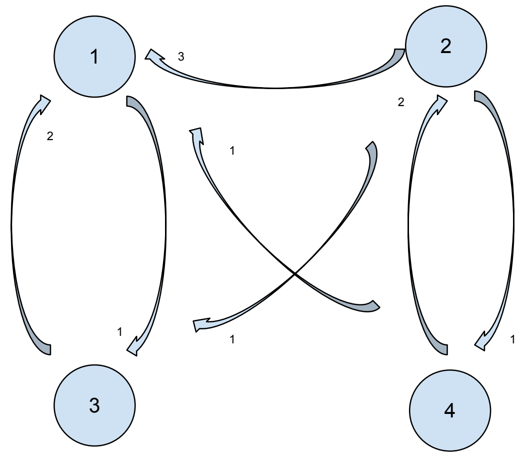
$$\begin{pmatrix}-1 & 1 & 0 & 0\\ 0 & -4 & 2 & 2\\ 1 & 0 & -2 & 1\\ 2 & 0 & 0 & -2\end{pmatrix}$$
(e)

$$\begin{pmatrix}0 & 0 & 0\\ 5 & -10 & 5\\ 10 & 0 & -10\end{pmatrix}$$
(f)

$$\begin{pmatrix}-.5 & .5 & 0 & 0\\ 0 & -.5 & .5 & 0\\ 0 & 0 & -.5 & .5\\ .5 & 0 & 0 & -.5\end{pmatrix}$$
(g)

$$\begin{pmatrix}\alpha & \beta\\ \omega & \gamma\end{pmatrix}$$
(h)

5. Consider the following continuous Markov chain.



(a) Obtain the transition rate matrix.

(b) Obtain the steady state probabilities for this Markov chain.

(c) Obtain the corresponding discrete time Markov chain.

(d) Draw the corresponding Markov chain.

(e) Obtain the steady state probabilities for the discretized Markov chain.